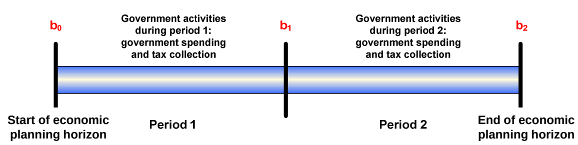
Government and Fiscal Policy in the Two-Period Framework (Continued)

October 10, 2011

A Dynamic Model of the Government

- So far only consumers in our two-period framework
- Introduce government in very simple form
  - Exists for both periods
  - Has spending in each period it needs to finance – can be financed via
    - Taxes
    - Issuing government debt / assets



- Notation
  - g_1: real government spending in period 1
  - g_2: real government spending in period 2
  - b_0: government asset position at beginning of period 1 / end of period 0
  - b_1: government asset position at beginning of period 2 / end of period 1
  - b_2: government asset position at beginning of period 3 / end of period 2
  - r: real interest rate between periods

Government Budget Constraint(s)

- Adopt a lifetime view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 government budget constraint $g_1 + b_1 = t_1 + (1+r)b_0$
  - Period-2 government budget constraint $g_2 + \cancel{b_2} = t_2 + (1+r)b_1$

Asset position at end of period 1 / beginning of period 2 the key link

Assume = 0 (no defaults + strictly increasing "utility")

  - Combine into lifetime budget constraint (LBC)
    - Solve period-2 budget constraint for b_1…
    - …and substitute into period-1 budget constraint

$$\underbrace{g_1 + \frac{g_2}{1+r}}_{\text{Present discounted value (PDV) of all lifetime government expenditure}} = \underbrace{t_1 + \frac{t_2}{1+r} + (1+r)b_0}_{\text{Present discounted value (PDV) of all lifetime government income}}$$

IMPORTANT: Government must balance its budget over its *lifetime*, not necessarily in each period

For graphical simplicity, will often assume $b_0 = 0$ (i.e., government begins life with zero net wealth). Note this is a *different* assumption than $b_2 = 0$.

Consumer Budget Constraint(s)

- Introduce tax payments into consumer side of framework
  - All in real terms for simplicity – can cast in nominal terms by multiplying by *P*
  - Period-1 budget constraint $c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$
  - Period-2 budget constraint $c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$
  - Combine into lifetime budget constraint (LBC)
    - Solve period-2 budget constraint for a_1…
    - …and substitute into period-1 budget constraint

$$\underbrace{c_1 + \frac{c_2}{1+r}}_{\text{Present discounted value (PDV) of all lifetime expenditure}} = \underbrace{y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0}_{\text{Present discounted value (PDV) of all lifetime disposable income (i.e., after-tax income)}}$$

Economy-Wide Resource Frontier

- **Consumer lifetime budget constraint**

$$c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} + (1+r)a_0$$

- **Government lifetime budget constraint**

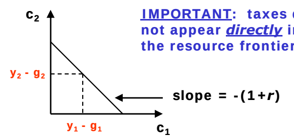
$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0$$

- **Summing the two yields economy-wide resource frontier**

$$c_1 + \frac{c_2}{1+r} = y_1 - g_1 + \frac{y_2 - g_2}{1+r} + (1+r)\cancel{(a_0 + b_0)}$$

Suppose = 0 for graphical simplicity

  - **aka "production possibilities frontier" (PPF)**
  - **The GDP accounting equation in two-period form**



Resource Frontier

IMPORTANT: taxes do not appear *directly* in the resource frontier…

THEOREM (intermediate micro): If taxes are *lump-sum*, then consumer optimal choices can be obtained by analyzing *either* the consumer LBC *or* the economy-wide resource frontier (superimpose indifference map), and either approach will yield the same predictions.

An important theoretical result for the analysis of tax policy.

Effects of Tax Policy

- **National savings = savings by consumers + savings by government + savings by firms**
  - **No firms in our model (yet..), so** $s_1^{firm} = 0$
  - $s_1^{priv} = y_1 - t_1 - c_1$
  - $s_1^{govt} = t_1 - g_1$
  - $s_1^{nat} = s_1^{priv} + s_1^{govt} = y_1 - \cancel{t_1} - c_1 + \cancel{t_1} - g_1 = y_1 - c_1 - g_1$

- **Policy Experiment: Is national savings affected by a decrease in t_1?**
  - **Suppose g_1 and g_2 do not change**
  - **Question 1: Effect on t_2?**
  - **Question 2: Effect of tax changes on consumers' optimal choice of period-1 consumption?**

Crucial logic
    - **Using intermediate micro theorem, NO EFFECT ON optimal c_1**
      - **Taxes are lump sum**
      - **Economy-wide resource constraint does not *directly* depend on taxes → optimal choice of c_1 unaffected by the change in tax policy**

EFFECTS OF TAX POLICY

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Analyzing effects of changes in tax policy on optimal consumption choices is the key

- Policy Experiment: Is national savings affected by a decrease in t_1?
  - Suppose g_1 and g_2 do not change
  - Question 1: Effect on t_2?
  - Question 2: Effect of tax changes on consumers' optimal choice of period-1 consumption?

Crucial logic
    - Using intermediate micro theorem, NO EFFECT ON optimal c_1
      - Taxes are lump sum
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  - Question 3: Effect of tax changes on period-1 national savings?
    - NONE – because neither g_1 nor c_1 changed

RICARDIAN EQUIVALENCE

- Ricardian Equivalence Theorem: For a given PDV of government spending, neither consumption nor national savings is affected by the precise timing of lump-sum taxes

- A benchmark result/concept in the theory of macroeconomic policy

- Economic Interpretation: Rational consumers understand that a tax cut in short run means a tax increase in the future (because PDV of government spending is unchanged)
  - Thus entire tax cut is saved by consumers in order to pay higher taxes in the future
  - Private savings and government savings move in exactly offsetting ways

- Analytically: key is that fiscal policy does not affect *real* i.r.

- Ricardian Equivalence is to tax theory as perfect competition is to basic economic theory
  - Prediction relies crucially on lump-sum taxes

NATURE OF TAXATION

- **Lump-Sum Tax**
  - **A tax whose total incidence (i.e., total amount paid) does not depend in any way on any economic decisions/choices an individual makes**
  - **Real-world examples: ?...**
- **Taxes in our two-period framework so far**
  - **Lump-sum! Total amounts t_1 and t_2 paid by consumer are independent of any of their decisions/choices**

Period-1 budget constraint

$$c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$$

Period-2 budget constraint

$$c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$$

NATURE OF TAXATION

- **Lump-Sum Tax**
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  - **Real-world examples: ?...**
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$$c_1 + t_1 + a_1 - a_0 = y_1 + ra_0$$

Period-2 budget constraint

$$c_2 + t_2 + a_2 - a_1 = y_2 + ra_1$$

- **Proportional (aka distortionary) Tax**
  - **A tax whose total incidence depends on economic decisions/choices an individual makes**
  - **In simple two-period framework: consumers only make consumption choices c_1 and c_2**

τ is consumption tax *rate* (aka sales tax rate)

Period-1 budget constraint

$$(1+\tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$$

Period-2 budget constraint

$$(1+\tau_2)c_2 + a_2 - a_1 = y_2 + ra_1$$

PROPORTIONAL TAXATION

τ is consumption tax rate (aka sales tax rate)

Period-1 budget constraint

$$(1+\tau_1)c_1 + a_1 - a_0 = y_1 + ra_0$$

Period-2 budget constraint

$$(1+\tau_2)c_2 + a_2 - a_1 = y_2 + ra_1$$

- **Combine into consumer LBC**

$$(1+\tau_1)c_1 + \frac{(1+\tau_2)c_2}{1+r} = y_1 + \frac{y_2}{1+r} + (1+r)a_0$$

- **Slope is** $-\left(\frac{1+\tau_1}{1+\tau_2}\right)(1+r)$

- **Non-lump-sum taxes: optimal consumption choices must be determined using consumer LBC, not economy's resource frontier (i.e., intermediate micro theorem does not apply)**

- **Changes in tax *rates* do affect optimal consumption choices because they change slope ("effective *real* i.r!") of consumer LBC**

- **Ricardian Equivalence Theorem does not apply**
  - **Changes in tax rates *do* affect national savings**

RICARDIAN EQUIVALENCE?

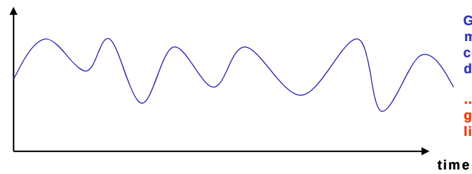
- **So why the fascination with Ricardian Equivalence?**

- **A benchmark result/concept in the theory of macroeconomic policy**
  - **Effects of actual policy proposals can be compared to the Ricardian Equivalence benchmark**
  - **In practice, *does* seem like tax rebates are sometimes saved**

RICARDIAN EQUIVALENCE?

- So why the fascination with Ricardian Equivalence?

- A benchmark result/concept in the theory of macroeconomic policy
  - Effects of actual policy proposals can be compared to the Ricardian Equivalence benchmark
  - In practice, ***does*** seem like tax rebates are sometimes saved
- At aggregate level, total tax collections sometime "seem" lump-sum (i.e., independent of aggregate macroeconomic activity)



GDP (or any aggregate measure of real activity…ie, consumption) moves up and down over time…

…but total tax revenue of the government fluctuates very little

- Ricardian Equivalence
  - Is a theoretical benchmark
  - Is an empirical benchmark

Ricardian Equivalence is about the (lack of) effects of ***changes in tax policy***, holding total government liabilities fixed. If g_1 and/or g_2 change, Ric. Equiv. does ***not*** apply.

INFINITE-PERIOD CONSUMER ANALYSIS

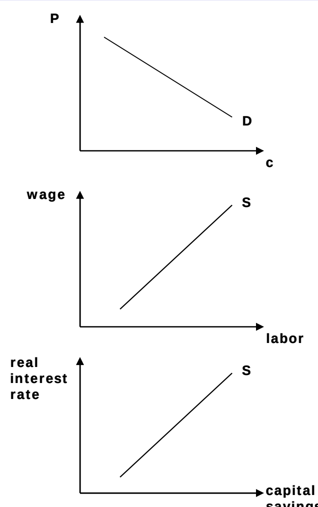
OCTOBER 10, 2011

Basics

- Modern workhorse macroeconomic models feature an **infinite** number of periods
    - A more realistic (?) view of time
- Especially useful for thinking about asset accumulation and asset pricing
    - The intersection of modern macro theory and modern finance theory
- Here, assume just one **real** asset
    - Call it a "stock" – i.e., a share in the S&P 500
    - (In Chapter 14, two nominal assets: bonds and money)
- Index time periods by arbitrary indexes *t*, *t*+1, *t*+2, etc.
    - Important: all analysis conducted from the perspective of the very beginning of period *t*…
    - …so an "infinite future" (period *t*+1, period, *t*+2, period *t*+3, …) for which to save

The Three Macro (Aggregate) Markets

- **Goods Markets**
    - Demand derived from C-L framework
- **Labor Markets**
    - Supply derived from C-L framework
- **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**
    - Supply derived from C-S framework

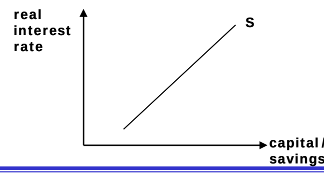


THE THREE MACRO (AGGREGATE) MARKETS

Many **different** ***types*** **of assets exist…hence many different** ***types of financial markets***

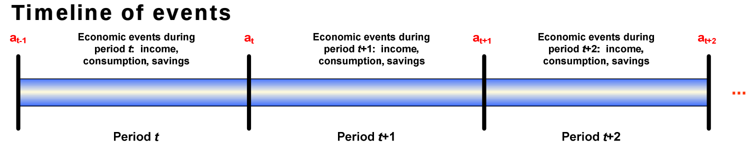
1. **Stock markets (Chapter 8)**
2. **Real-estate markets (Problem Set 5)**
3. **Bond markets (Chapter 14)**
4. **Money markets (Chapter 14)**

- **Capital / Savings / Funds / Asset Markets (aka Financial Markets)**
  - **Supply derived from C-S framework**



BASICS

- **Timeline of events**



- **Notation**
  - c_t: **consumption in period** t
  - P_t: **nominal price of consumption in period** t
  - Y_t: **nominal income in period** t **("falls from the sky")**
  - a_{t-1}: **real wealth (stock) holdings at beginning of period** t **/ end of period** t**-1**
  - S_t: **nominal price of a unit of stock in period** t
  - D_t: **nominal dividend paid in period** t **by each unit of stock held at the** **<u>start</u>** **of** t

The "definining features" of stock

  - π_{t+1}: **net inflation rate between period** t **and period** t**+1**

$$\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \left(= \frac{P_{t+1}}{P_t} - 1 \right)$$

  - y_t: **real income in period** t **(=** Y_t / P_t**)**

BASICS

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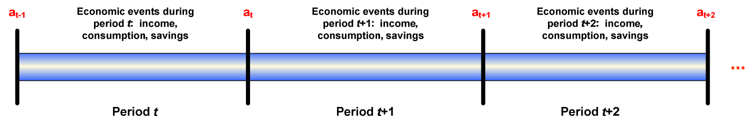

- **Notation**
  - c_{t+1}: consumption in period $t+1$
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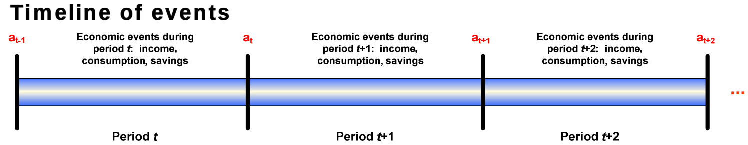
  - Y_{t+1}: nominal income in period $t+1$ ("falls from the sky")
  - a_t: real wealth (stock) holdings at beginning of period $t+1$ / end of period t
  - The "definining features" of stock:
    - S_{t+1}: nominal price of a unit of stock in period $t+1$
    - D_{t+1}: nominal dividend paid in period t by each unit of stock held at the start of $t+1$
  - π_{t+2}: net inflation rate between period $t+1$ and period $t+2$

$$\pi_{t+2} = \frac{P_{t+2} - P_{t+1}}{P_{t+1}} \left(= \frac{P_{t+2}}{P_{t+1}} - 1 \right)$$

  - y_{t+1}: real income in period $t+1$ ($= Y_{t+1} / P_{t+1}$)

BASICS

- **Timeline of events**



- **Notation**
  - And so on for period $t+2$, $t+3$, etc…

SUBJECTIVE DISCOUNT FACTOR

- **Infinite number of periods a more serious view of time**
- **Impatience potentially an issue when taking a serious view of time**
- **Individuals (i.e., consumers) are impatient**
  - All else equal, would rather have experience X utils today than identical X utils at some future date
  - An introspective statement about the world
  - An empirical statement about the world

Macro Fundamentals

SUBJECTIVE DISCOUNT FACTOR

- **Infinite number of periods a more serious view of time**
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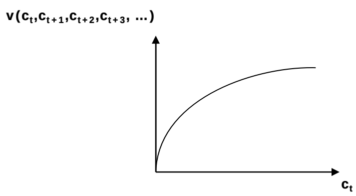
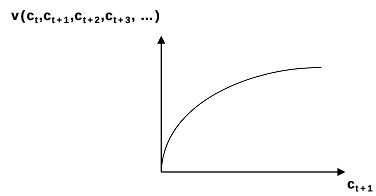
- **Individuals (i.e., consumers) are impatient**
  - All else equal, would rather have experience X utils today than identical X utils at some future date
  - An introspective statement about the world
  - An empirical statement about the world
- **Subjective discount factor**
  - A simple model of consumer impatience
  - β (a number between zero and one) measures impatience
    - The lower is β, the less does individual value future utility
  - Simple assumption about how "impatience" builds up over time
    - Multiplicatively: i.e., discount one period ahead by β, discount two periods ahead by β^2, discount three periods ahead by β^3, etc.
    - Do individuals' impatience really build up over time in this way?...limited empirical evidence so really don't know…

UTILITY

- Preferences $v(c_t, c_{t+1}, c_{t+2}, \ldots)$ with all the "usual properties"
  - Lifetime utility function
  - Strictly increasing in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$
  - Diminishing marginal utility in $c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots$





etc.

Model Structure

UTILITY

- Preferences $v(c_t, c_{t+1}, c_{t+2}, \ldots)$ with all the "usual properties"
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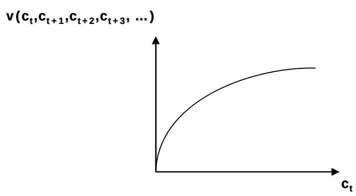

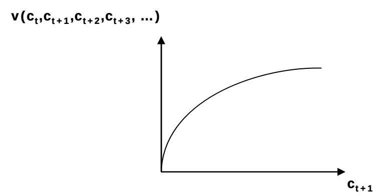

etc.

- Lifetime utility function additively-separable across time (a simplifying assumption), starting at time *t*

$$v(c_t, c_{t+1}, c_{t+2}, c_{t+3}, \ldots) = u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \ldots$$

- **Utility side of infinite-period model no different than Chapter 1 model – except no longer possible to represent graphically**

BUDGET CONSTRAINT(S)

- **Suppose again *Y* "falls from the sky"**
  - **Y_t in period *t*, Y_{t+1} in period *t*+1, Y_{t+2} in period *t*+2, etc.**
- **Need infinite budget constraints to describe economic opportunities and possibilities**
  - **One for each period**
  - **Period-*t* budget constraint**

$$\underbrace{P_t c_t + S_t a_t}_{\text{Total expenditure in period } t} = \underbrace{Y_t + S_t a_{t-1} + D_t a_{t-1}}_{\text{Total income in period } t}$$

Total expenditure in period *t*: period-*t* consumption + wealth to *carry into period t*+1

Total income in period *t*: period-*t* *Y* + income from stock-holdings *carried into period t* (has value S_t and pays dividend D_t)

  - **Period *t*+1 budget constraint**

$$\underbrace{P_{t+1} c_{t+1} + S_{t+1} a_{t+1}}_{\text{Total expenditure in period } t+1} = \underbrace{Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t}_{\text{Total income in period } t+1}$$

Total expenditure in period *t*+1: period-*t*+1 consumption + wealth to *carry into period t*+2

Total income in period *t*+1: period-*t*+1 *Y* + income from stock-holdings *carried into period t*+1 (has value S_{t+1} and pays dividend D_{t+1})

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Total expenditure in period *t*+1: period-*t*+1 consumption + wealth to *carry into period t*+2

Total income in period *t*+1: period-*t*+1 *Y* + income from stock-holdings *carried into period t*+1 (has value S_{t+1} and pays dividend D_{t+1})

And identical-looking budget constraints for *t*+2, *t*+3, *t*+4, etc…

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Sequential formulation highlights the role of stock holdings (a_t) between period t and period $t+1$**
  - Accords better with the explicit timing of economic events than the lifetime approach…
  - …but yields the same result
  - Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory)

- Apply Lagrange tools to consumption-savings optimization
- Objective function: $v(c_t, c_{t+1}, c_{t+2}, \ldots)$
- Constraints:
  - Period-t budget constraint: $Y_t + S_t a_{t-1} + D_t a_{t-1} - P_t c_t - S_t a_t = 0$
  - Period-$t+1$ budget constraint: $Y_{t+1} + S_{t+1} a_t + D_{t+1} a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} = 0$
  - Period-$t+2$ budget constraint: $Y_{t+2} + S_{t+2} a_{t+1} + D_{t+2} a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} = 0$
  - etc…

INFINITE constraints

- Sequential Lagrange formulation requires infinite multipliers

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1:** Construct Lagrange function (starting from t)

IMPORTANT: Discount factor β multiplies *both* future utility *and* future budget constraints

Everything (utility *and* income) about the future is discounted

$$
\begin{aligned}
&u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \ldots \\
&+\lambda_t \left[Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t\right] \\
&+\beta\lambda_{t+1} \left[Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1}\right] \\
&+\beta^2\lambda_{t+2} \left[Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2}\right] \\
&+\beta^3\lambda_{t+3} \left[Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3}\right] \\
&+\ldots
\end{aligned}
$$

First the lifetime utility function….

…then the period t constraint…

…then the period $t+1$ constraint…

…then the period $t+2$ constraint…

…then the period $t+3$ constraint…

Infinite number of terms

LAGRANGE ANALYSIS: SEQUENTIAL APPROACH

- **Step 1: Construct Lagrange function (starting from *t*)**

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Everything (utility ***and*** income) about the future is discounted

$$u(c_t)+\beta u(c_{t+1})+\beta^2 u(c_{t+2})+\beta^3 u(c_{t+3})+\ldots$$ First the lifetime utility function....

$$+\lambda_t\left[Y_t+(S_t+D_t)a_{t-1}-P_tc_t-S_ta_t\right]$$ ...then the period *t* constraint...

$$+\beta\lambda_{t+1}\left[Y_{t+1}+(S_{t+1}+D_{t+1})a_t-P_{t+1}c_{t+1}-S_{t+1}a_{t+1}\right]$$ ...then the period *t*+1 constraint...

$$+\beta^2\lambda_{t+2}\left[Y_{t+2}+(S_{t+2}+D_{t+2})a_{t+1}-P_{t+2}c_{t+2}-S_{t+2}a_{t+2}\right]$$ ...then the period *t*+2 constraint...

$$+\beta^3\lambda_{t+3}\left[Y_{t+3}+(S_{t+3}+D_{t+3})a_{t+2}-P_{t+3}c_{t+3}-S_{t+3}a_{t+3}\right]$$ ...then the period *t*+3 constraint...

$$+\ldots$$ Infinite number of terms

- **Step 2: Compute FOCs with respect to c_t, a_t, c_{t+1}, a_{t+1}, c_{t+2}, …**

with respect to c_t: Equation 1

with respect to a_t: Equation 2

with respect to c_{t+1}: Equation 3

THE BASICS OF ASSET PRICING

$$u'(c_t)-\lambda_tP_t=0$$ Equation 1

$$-\lambda_tS_t+\beta\lambda_{t+1}(S_{t+1}+D_{t+1})=0$$ Equation 2

$$u'(c_{t+1})-\lambda_{t+1}P_{t+1}=0$$ Equation 3

- **Equation 2 →** $$S_t=\left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1}+D_{t+1})$$ BASIC ASSET-PRICING EQUATION

Period-*t* stock price = Pricing kernel x Future return

- **Much of finance theory concerned with pricing kernel**
  - **Theoretical properties**
  - **Empirical models of kernels**
- **Pricing kernel where macro theory and finance theory intersect**
  - **Allows studying common "macro factors" that affect "all" asset markets/asset prices**
  - **Chapter 14: link between pricing kernel and bond prices**

THE BASICS OF ASSET PRICING

$$u'(c_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$
$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0 \quad \text{Equation 2}$$
$$u'(c_{t+1}) - \lambda_{t+1} P_{t+1} = 0 \quad \text{Equation 3}$$

- Equation 2 → $S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$ BASIC ASSET-PRICING EQUATION

Period-t stock price = Pricing kernel x Future return

Two components:
1. Future price of stock
2. Future dividend payment

- Much of finance theory concerned with pricing kernel
  - Theoretical properties
  - Empirical models of kernels
- Pricing kernel where macro theory and finance theory intersect
- Why buy an asset?
  - May pay a dividend in the future
  - Market value may rise in the future: $S_{t+1} / S_t > 1$ is "capital gain"

Finance Fundamentals

THE BASICS OF ASSET PRICING

$$u'(c_t) - \lambda_t P_t = 0 \quad \text{Equation 1}$$
$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0 \quad \text{Equation 2}$$
$$u'(c_{t+1}) - \lambda_{t+1} P_{t+1} = 0 \quad \text{Equation 3}$$

- Equation 2 → $S_t = \left(\frac{\beta\lambda_{t+1}}{\lambda_t}\right)(S_{t+1} + D_{t+1})$ BASIC ASSET-PRICING EQUATION

Period-t stock price = Pricing kernel x Future return

Two components:
1. Future price of stock
2. Future dividend payment

- Much of finance theory concerned with pricing kernel
  - Theoretical properties
  - Empirical models of kernels
- Pricing kernel where macro theory and finance theory intersect
- To take more macro-centric view
  - Solve equations 1 and 3 for λ_t and λ_{t+1}
  - Insert in asset-pricing equation

Macroeconomic Events Affect Asset Prices

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1}+D_{t+1})\left(\frac{P_t}{P_{t+1}}\right)$$

Using definition of inflation: $1+\pi_{t+1} = P_{t+1} / P_t$

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1}+D_{t+1})\left(\frac{1}{1+\pi_{t+1}}\right)$$

VIEW AS A CONSUMPTION-SAVINGS OPTIMALITY CONDITION

- Consumption across time (c_t and c_{t+1}) affects stock prices
  - Fluctuations over time in aggregate consumption impact S_t
- Inflation affects stock prices
  - Fluctuations over time in inflation impact S_t
- ANY factor (monetary policy, fiscal policy, globalization, etc.) that affects inflation and GDP in principle impacts stock/asset markets
- Direction of causality?...

Consumer Optimization

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1}+D_{t+1})\left(\frac{P_t}{P_{t+1}}\right)$$

Move $u'(c_t)$ and $\beta u'(c_{t+1})$ terms to left-hand-side, and S_t to right-hand-side

$$\underbrace{\frac{u'(c_t)}{\beta u'(c_{t+1})}}_{\text{MRS between period } t \text{ consumption and period } t+1 \text{ consumption}} = \underbrace{\left(\frac{S_{t+1}+D_{t+1}}{S_t}\right)\left(\frac{1}{1+\pi_{t+1}}\right)}_{\text{Some sort of price ratio...}}$$

i.e., ratio of marginal utilities

Consumer Optimization

$$S_t = \left(\frac{\beta u'(c_{t+1})}{u'(c_t)}\right)(S_{t+1}+D_{t+1})\left(\frac{P_t}{P_{t+1}}\right)$$

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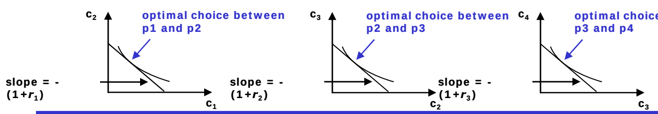
CONSUMPTION-SAVINGS OPTIMALITY CONDITION

i.e., ratio of marginal utilities → MRS between period t consumption and period $t+1$ consumption

Recall real interest rate is a ***price*** → Analogy with Chapters 3 & 4: must be $(1+r_t)$

- Recover Chapter 3 & 4 framework by setting t =1 and β = 1
- Infinite-period framework is sequence of overlapping two-period frameworks



etc.

A Long-Run Theory of Macro

- Consumption-savings optimality condition at the heart of modern macro models
  - Emphasize the dynamic nature of aggregate economic events
  - Foundation for understanding the periodic ups and downs ("business cycles") of the economy
  - (Chapter 13: business cycle theories)

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = 1 + r_t$$

NEXT: Impose "steady state" and examine long-run relationship between interest rates and consumer impatience

$$\frac{1}{\beta} = 1 + r$$