FIRMS IN THE TWO-PERIOD FRAMEWORK

MARCH 12, 2012

BASICS

- ☐ **Embed firms in two-period (multi-period) economy**
- ☐ **In each period *t*, representative firm produces according to a production technology $A_t f(k_t, n_t)$**
  - ☐ **n_t: labor used for production**
  - ☐ **k_t: capital ("machines and equipment") used for production**
  - ☐ **A_t: total factor productivity**
    - ☐ **A catch-all measure for level of sophistication of technology**
    - ☐ **Real Business Cycle (RBC) view: the driving force behind the periodic ups and downs of macroeconomic activity (Chapter 13)**
    - ☐ **For now, suppose $A_t = 1$ always (i.e., in both period 1 and 2)**

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- **Broad macro view of the factors of production**
  - **Labor – all types**
  - **Capital**
    - **Machines and equipment**
    - **Trucks**
    - **Factories**
    - **An accumulation (i.e., stock) variable, NOT a flow**
    - **Takes time to build capital (simple starting assumption: takes one period)**

Can also think of education and other intangibles (i.e., experience, brand name) as "capital"

The function $f(k, n)$ describes how capital and labor combine with each other to yield output (goods)

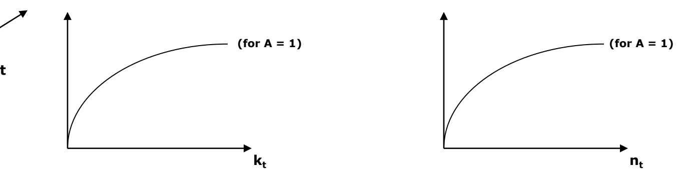
PRODUCTION FUNCTION

- **Production function $f(k_t, n_t)$ with all the "usual properties" of production functions**
  - **Strictly increasing in k_t and n_t**
  - **Diminishing marginal product in k_t and n_t**

Recall from basic micro

The extra output that results from using one additional unit of input

total output (i.e., GDP)



for any *t*

Model Structure

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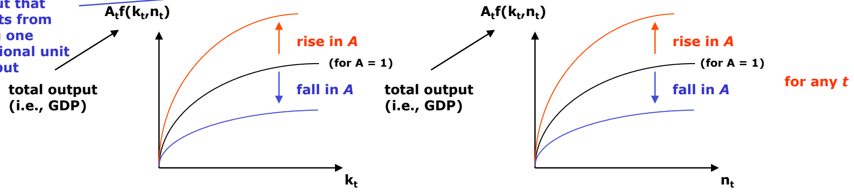

- ❑ **When allow time-varying A_t (Chapter 9 and Chapter 13), changes in A cause shifts in production function**
  - ❑ **Source of business cycle fluctuations in RBC theory**

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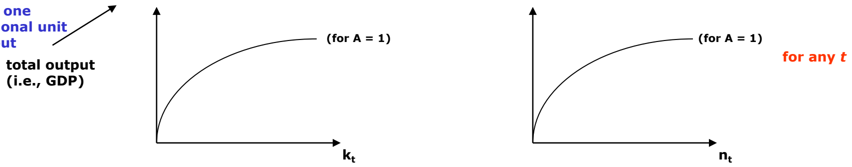
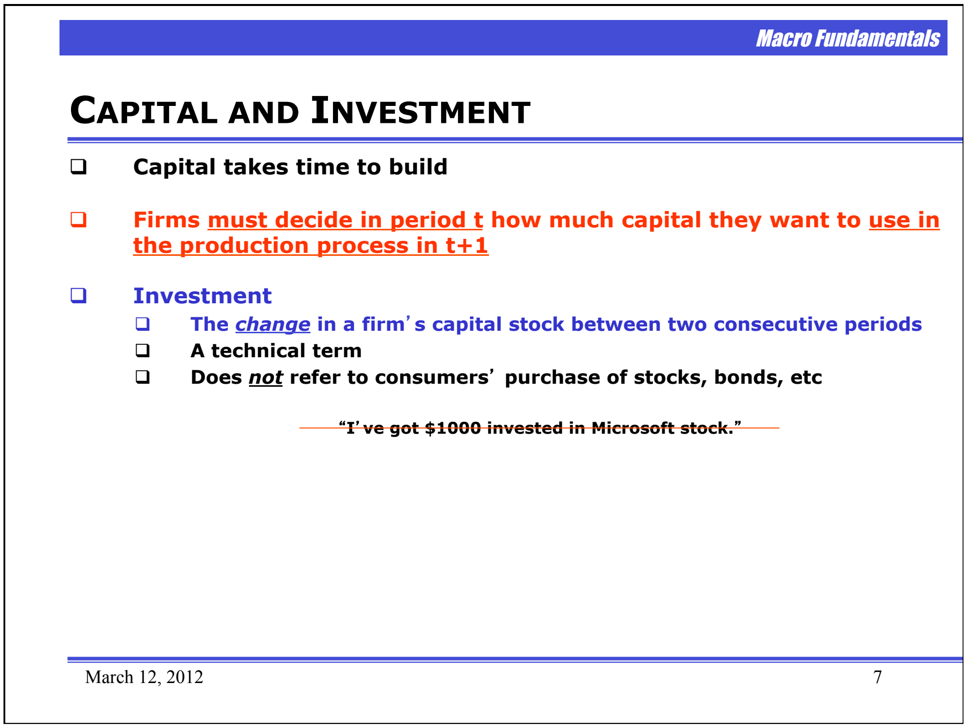


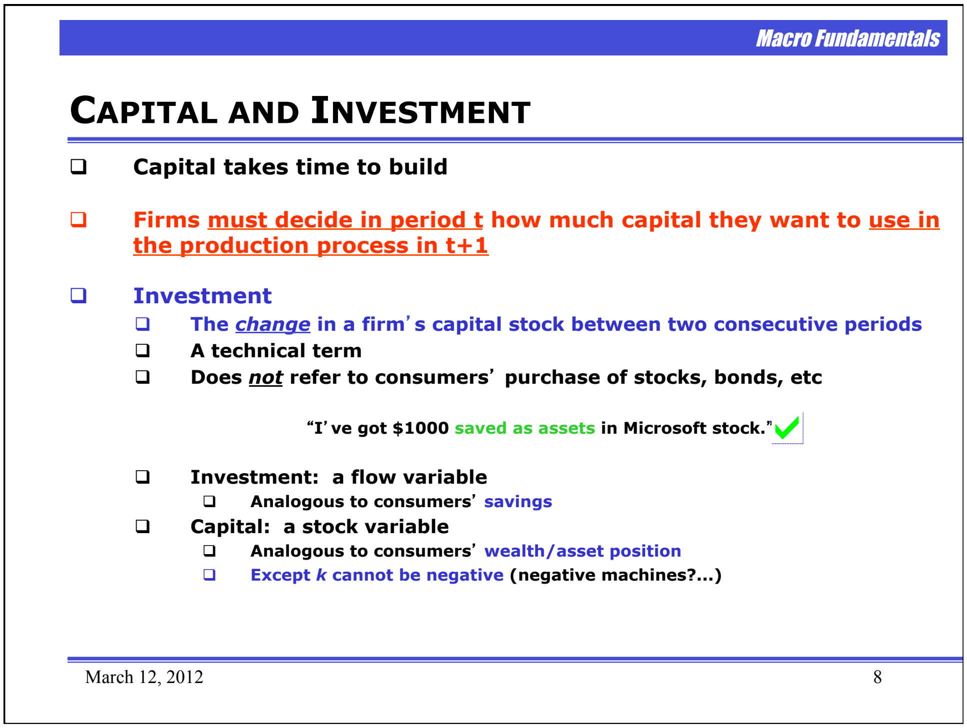
- ❑ **When allow time-varying A_t (Chapter 9 and Chapter 13), changes in A cause shifts in production function**
  - ❑ **Source of business cycle fluctuations in RBC theory**
- ❑ **For now suppose $A_t = 1$ in each period**



CAPITAL AND INVESTMENT

- ❑ **Capital takes time to build**
- ❑ **Firms <u>must decide in period t</u> how much capital they want to <u>use in the production process in t+1</u>**
- ❑ **Investment**
  - ❑ **The *<u>change</u>* in a firm's capital stock between two consecutive periods**
  - ❑ **A technical term**
  - ❑ **Does *<u>not</u>* refer to consumers' purchase of stocks, bonds, etc**

~~**"I've got $1000 invested in Microsoft stock."**~~



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  - ❑ **Investment: a flow variable**
    - ❑ **Analogous to consumers' savings**
  - ❑ **Capital: a stock variable**
    - ❑ **Analogous to consumers' wealth/asset position**
    - ❑ **Except *k* cannot be negative (negative machines?...)**

Macro Fundamentals

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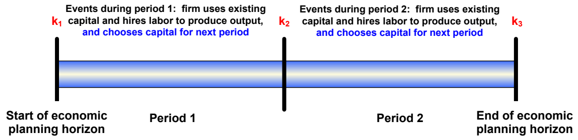
    - ❑ **Except *k* cannot be negative (negative machines?...)**
  - ❑ **One of the components of GDP (= C + I + G + NX)**
    - ❑ **Investment comprises ≈ 15% of GDP in U.S.**
    - ❑ **Investment comprises ≈ 40% of GDP in China (high I drives rapid growth)**

Model Structure

BASICS

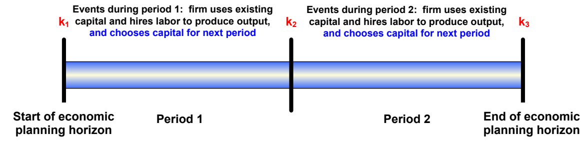
- ❑ **Timeline of events**



- ❑ **Notation**
  - ❑ k_1: **capital used for production in period 1 (decided upon in "period 0")**
  - ❑ n_1: **labor used for production in period 1**
  - ❑ w_1: **real wage rate for labor in period 1 ($w_1 = W_1/P_1$)**
  - ❑ i: **nominal interest rate**
  - ❑ P_1: **nominal price of output produced and sold by firm in period 1**

BASICS

☐ **Timeline of events**



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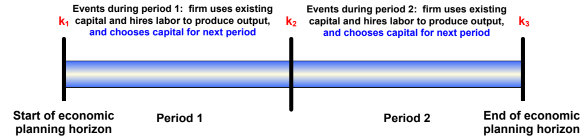
- ☐ P_1: nominal price of output produced and sold by firm in period 1 AND nominal price of one unit of capital bought by the firm in period 1 for use in period 2 (recall time to build…)

Underlying assumption/view of world: *capital goods* are not necessarily "distinct" from *consumption goods* (i.e., computers purchased by both firms and individual consumers)

BASICS

☐ **Timeline of events**



☐ **Notation**

- ☐ k_2: capital used for production in period 2 (decided upon in period 1)
- ☐ n_2: labor used for production in period 2
- ☐ w_2: real wage rate for labor in period 2 ($w_2 = W_2/P_2$)
- ☐ i: nominal interest rate
- ☐ P_2: nominal price of output produced and sold by firm in period 2 AND nominal price of one unit of capital bought by the firm in period 2 for use in period 3 (recall time to build…)

Underlying assumption/view of world: *capital goods* are not necessarily "distinct" from *consumption goods* (i.e., computers purchased by both firms and individual consumers)

FIRM PROFIT MAXIMIZATION

- **A *dynamic* profit maximization problem**
  - **Because firm exists for both periods**
  - **All analysis conducted from the perspective of the very beginning of period 1**
  - **→ Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits**

- **Dynamic profit function**
  - **(specified in nominal terms – could specify in real terms…)**

$$\overbrace{\underbrace{P_1 f(k_1,n_1)}_{\text{Total revenue in period 1 (price x output)}} + \underbrace{P_1k_1}_{\text{Value of pre-existing capital (an asset for firms)}} - \underbrace{P_1w_1n_1}_{\text{Total labor cost in period 1}} - \underbrace{P_1k_2}_{\text{Total cost of buying capital for period 2 (time to build} \rightarrow \text{must purchase period-2 capital in period 1)}}}^{\text{Period-1 profits}}$$

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- **Two-period framework: k_3 = 0 (no machines needed in "period 3")**

FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1,n_1) + P_1 k_1 - P_1 w_1 n_1 - P_1 k_2 + \frac{P_2 f(k_2,n_2)}{1+i} + \frac{P_2 k_2}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 \cancelto{= 0}{k_3}}{1+i}$$

- ☐ **FOCs with respect to n_1, n_2, k_2**

Identical except for time subscripts

with respect to n_1: $\cancel{P_1} f_n(k_1,n_1) - \cancel{P_1} w_1 = 0$ **Equation 1**

with respect to n_2: $\frac{\cancel{P_2} f_n(k_2,n_2)}{\cancel{1+i}} - \frac{\cancel{P_2} w_2}{\cancel{1+i}} = 0$ **Equation 2**

with respect to k_2: $-P_1 + \frac{P_2 f_k(k_2,n_2)}{1+i} + \frac{P_2}{1+i} = 0$ **Equation 3**

FIRM PROFIT MAXIMIZATION

- ☐ **Re-express equation 3**

$$-P_1 + \frac{P_2 f_k(k_2,n_2)}{1+i} + \frac{P_2}{1+i} = 0 \xrightarrow{\text{Divide by } P_1} \frac{P_2 f_k(k_2,n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} = 1$$

Group terms informatively

$$\longrightarrow \left(\frac{P_2}{P_1}\right)\left(\frac{1}{1+i}\right) f_k(k_2,n_2) + \left(\frac{P_2}{P_1}\right)\left(\frac{1}{1+i}\right) = 1 \xrightarrow{P_2/P_1 = 1 + \pi_2} \left(\frac{1+\pi_2}{1+i}\right) f_k(k_2,n_2) + \left(\frac{1+\pi_2}{1+i}\right) = 1$$

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$$\xrightarrow{\quad} \left(\frac{P_2}{P_1}\right)\left(\frac{1}{1+i}\right) f_k(k_2,n_2) + \left(\frac{P_2}{P_1}\right)\left(\frac{1}{1+i}\right) = 1 \xrightarrow{P_2/P_1 \,=\, 1 + \pi_2} \left(\frac{1+\pi_2}{1+i}\right) f_k(k_2,n_2) + \left(\frac{1+\pi_2}{1+i}\right) = 1$$

Fisher equation

$$\xrightarrow{\quad} \frac{f_k(k_2,n_2)}{1+r} + \frac{1}{1+r} = 1 \xrightarrow{\text{Multiply by } 1+r} f_k(k_2,n_2) + \cancel{1} = \cancel{1} + r$$

$$\xrightarrow{\quad} \boxed{f_k(k_2,n_2) = r}$$

Equivalent/alternative representation of firm profit-maximizing condition for capital

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with respect to *k*$_2$: $-P_1 + \frac{P_2 f_k(k_2,n_2)}{1+i} + \frac{P_2}{1+i} = 0 \xleftrightarrow{\text{equivalent}} f_k(k_2,n_2) = r$ **Equation 3**

☐ **Profit-maximizing labor hiring implies**

$f_n(k_1,n_1) = w_1$ **AND** $f_n(k_2,n_2) = w_2$

☐ **Profit-maximizing capital purchases (for the future…) implies**

$$f_k(k_2,n_2) = r$$

Model Structure

FIRM PROFIT MAXIMIZATION

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- ❑ **Marginal product of labor**
  - ❑ **$f_n(k_t,n_t)$**
  - ❑ **Sometimes denote by mpn_t**
- ❑ **Marginal product of capital**
  - ❑ **$f_k(k_t,n_t)$**
  - ❑ **Sometimes denote by mpk_t**

These FOCs are foundation for:

1. **Labor Demand**
2. **Capital/Investment Demand**

Macro Fundamentals

COBB-DOUGLAS PRODUCTION FUNCTION

- ❑ **A commonly-used functional form in modern quantitative macroeconomic analysis**

$$f(k_t,n_t) = k_t^{\alpha} n_t^{1-\alpha}$$

(saw Cobb-Douglas utility function on Problem Set 1)

- ❑ **Describes the empirical relationship between aggregate GDP, aggregate capital, and aggregate labor quite well**
- ❑ $\alpha \in (0,1)$ **measures capital's share of output**
  - ❑ **Hence** $(1-\alpha) \in (0,1)$ **measures labor's share of output**
  - ❑ **Interpretation**
    - ❑ **The relative importance of (either) capital (or labor) in the production process**

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  - ❑ **Interpretation**
    - ❑ **The relative importance of (either) capital (or labor) in the production process**
  - ❑ **Estimates for U.S. economy:** $\alpha \approx 0.3$
  - ❑ **Estimates for Chinese economy:** $\alpha \approx 0.15$ **(not (yet) a very capital-rich economy)**
- ❑ **Cobb-Douglas form useful for illustrating factor demands**
  - ❑ $mpn_t = f_n(k_t, n_t) = (1-\alpha)k_t^{\alpha} n_t^{-\alpha}$
  - ❑ $mpk_t = f_k(k_t, n_t) = \alpha k_t^{\alpha-1} n_t^{1-\alpha}$

MICRO-LEVEL LABOR DEMAND

- ❑ **Firm-level demand for labor defined by the relation**

Follows from Equation 1 and Equation 2

$$w_t = (1-\alpha)k_t^{\alpha} n_t^{-\alpha} (= mpn_t)$$

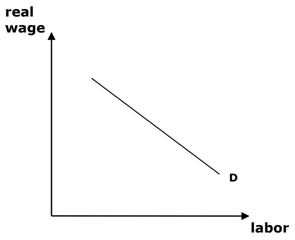
for both t = 1 and t = 2

$$\Big\downarrow$$

Because exponent (-a) is a negative number, can move to denominator

$$w_t = (1-\alpha)\left(\frac{k_t}{n_t}\right)^{\alpha}$$

? RELATIONSHIP BETWEEN w_t and n_t



Labor Demand in the Micro and the Macro

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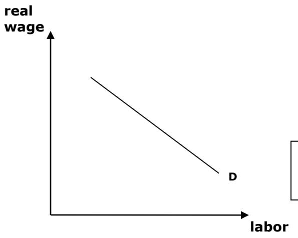
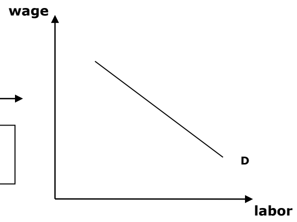


Firm-level labor demand function

Sum over all firms

(No tension between the micro and macro as there is for labor supply)



Aggregate-level labor demand function

☐ **Completes picture of the aggregate labor market**

Capital Demand in the Micro

MICRO-LEVEL CAPITAL DEMAND

☐ **Firm-level demand for capital defined by the relation**

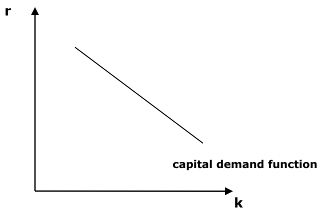
Follows from Equation 3 (will see it soon…)

$$r_t = \alpha k_t^{\alpha-1}n_t^{1-\alpha}(= mpk_t)$$

Because exponent (*α* – 1) is a negative number, can move to denominator

$$r_t = \alpha\left(\frac{n_t}{k_t}\right)^{1-\alpha}$$

INVERSE RELATIONSHIP BETWEEN r_t and k_t



Capital Demand in the Micro and the Macro

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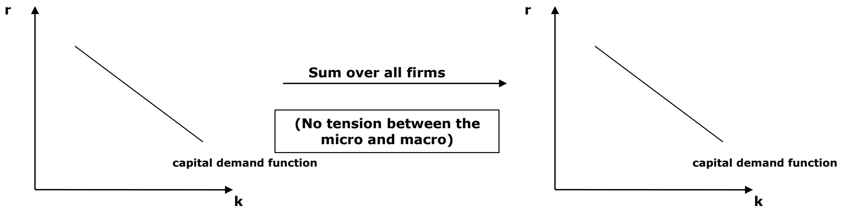


Sum over all firms

(No tension between the micro and macro)

Firm-level capital demand function

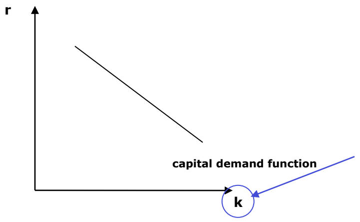
Aggregate-level capital demand function

- ❑ **(Almost…) completes picture of the aggregate capital market**

Investment Demand

FROM CAPITAL DEMAND TO INVESTMENT DEMAND

- ❑ **Capital is an accumulation (stock) variable**



Want investment (a flow) to show up here, not capital (a stock)

Investment is *change* in capital stock between consecutive periods

FROM CAPITAL DEMAND TO INVESTMENT DEMAND

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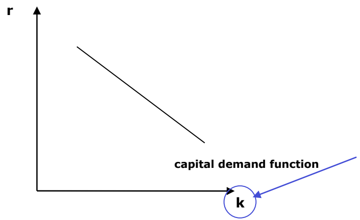

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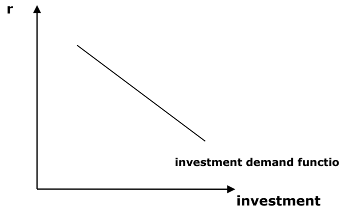
Investment is ***change*** in capital stock between consecutive periods

$inv_1 = k_2 - k_1$

At start of period 1, k_1 cannot be changed. Thus any rise in demand for k_2 is reflected one-for-one in a rise in inv_1.

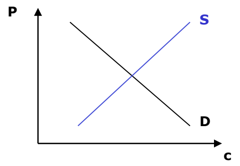
→ Capital demand and investment demand functions have same shape

- ❑ **Investment is a flow variable**

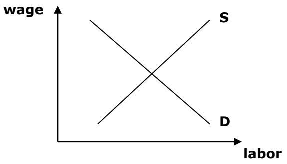


THE THREE MACRO (AGGREGATE) MARKETS

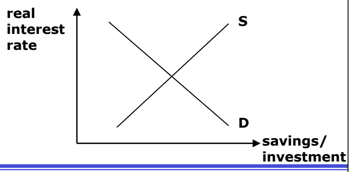
- ❑ **Goods Markets**
  - ❑ **Demand derived from C-L framework**
    (Supply: have to consider how aggregate NOMINAL P is determined…Chapter 14)



- ❑ **Labor Markets**
  - ❑ **Supply derived from C-L framework**
  - ❑ **Demand derived from firm theory in C-L framework**



- ❑ **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**
  - ❑ **Supply derived from C-S framework**
    Demand derived from firm theory in C-S framework



REAL INTEREST RATE

- ***r*** **a key variable for macroeconomic analysis**
- **Chapter 4:** ***r*** **measures the price of period-1 consumption in terms of period-2 consumption**
- **Chapter 8:** ***r*** **reflects degree of impatience (in the long run)**
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- **Now: *r* measures the price of capital (machine and equipment) purchases by firms**
  - Reflects (real!) opportunity cost of sinking funds into capital *today* that won't bear fruit (i.e., help produce output) until the *future*
    - Regardless of whether firm actually has to "borrow" to purchase capital
  - Can see it mathematically

$$-P_1 + \frac{P_2 f_k(k_2,n_2)}{1+i} + \frac{P_2}{1+i} = 0$$

Equation 3 (FOC on k_2)

$$\downarrow$$

$$\downarrow$$

$$\downarrow$$

$$f_k(k_2,n_2) = r$$

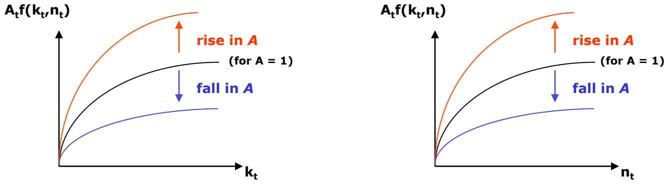
When firms make optimal investment decisions → $r = mpk$

SHOCKS

MARCH 12, 2012

BASICS

- **Shock/shifter**
  - **Definition: Some unexpected event that affects economic fundamentals and hence decisions, but which is unexplained or unexplainable**
- **Introducing shocks into our frameworks (consumption-leisure, consumption-savings, infinite-period) will "get them moving"**
- **Consider (for now) two types of shocks (one supply, one demand)**
  - **Total Factor Productivity (TFP) Shocks – unexpected changes in A_t in the production function $A_t f(k_t, n_t)$**



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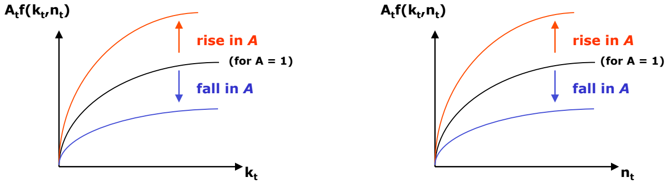

Changes in "consumer confidence"

  - **Preference Shocks – unexpected changes in representative consumer's utility function; causes rotations of indifference maps**

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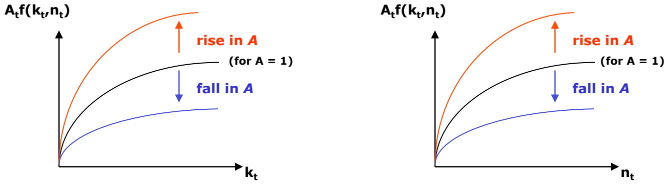
"SUPPLY SHOCK"

  - ❑ **Total Factor Productivity (TFP) Shocks – unexpected changes in A_t in the production function $A_tf(k_t, n_t)$**



"DEMAND SHOCK"

  - ❑ **Preference Shocks – unexpected changes in representative consumer's utility function; causes rotations of indifference maps**

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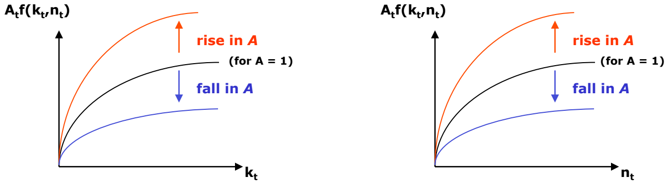
Later:

Monetary policy shocks

Fiscal policy shocks

Financial shocks



  - ❑ **Preference Shocks – unexpected changes in representative consumer's utility function; causes rotations of indifference maps**

TFP IN COBB-DOUGLAS PRODUCTION FUNCTION

- Revisit the commonly-used functional form in modern quantitative macroeconomic analysis

$$\text{output}_t = A_t f(k_t, n_t) = A_t k_t^{\alpha} n_t^{1-\alpha}$$

- Describes the empirical relationship between aggregate output, aggregate capital, aggregate labor, and level of sophistication of technology (TFP)
  - (How to measure TFP in Chapter 13)
- Cobb-Douglas form useful for illustrating effects of TFP shocks

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- Unexpected change (i.e., a shock) in A_t
  - Causes change in marginal product of labor

$$mpn_t = \frac{\partial \text{output}_t}{\partial n_t} = A_t f_n(k_t, n_t) = A_t (1-\alpha) k_t^{\alpha} n_t^{-\alpha}$$

Recall *mpn* is foundation for labor demand

  - Causes change in marginal product of capital

$$mpk_t = \frac{\partial \text{output}_t}{\partial k_t} = A_t f_k(k_t, n_t) = A_t \alpha k_t^{\alpha-1} n_t^{1-\alpha}$$

Recall *mpk* is foundation for capital/investment demand

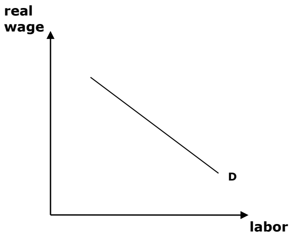
TFP SHOCKS AND LABOR DEMAND

- ☐ **Firm-level demand for labor defined by the relation**

$$w_t = A_t(1-\alpha)k_t^{\alpha}n_t^{-\alpha}(= mpn_t)$$

Because exponent (-α) is a negative number, can move to denominator

$$w_t = A_t(1-\alpha)\left(\frac{k_t}{n_t}\right)^{\alpha}$$



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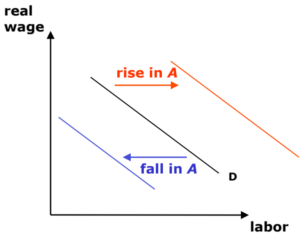

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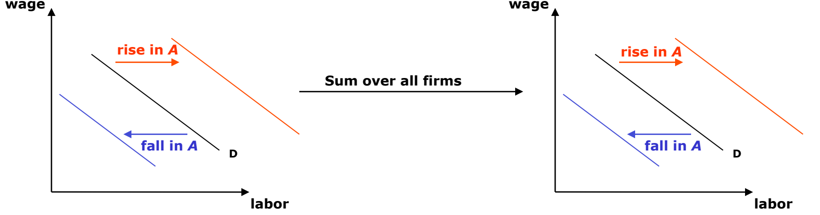


Firm-level labor demand function

Aggregate-level labor demand function

- **<u>IMPORTANT</u>: TFP shocks shift the labor demand curve**

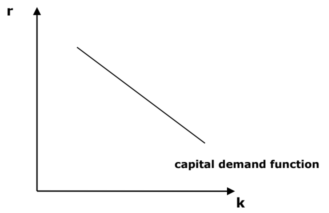
TFP SHOCKS AND CAPITAL DEMAND

- **Firm-level demand for capital defined by the relation**

$$r_t = A_t\alpha k_t^{\alpha-1}n_t^{1-\alpha}(= mpk_t)$$

$\downarrow$ **Because exponent ($\alpha - 1$) is a negative number, can move to denominator**

$$r_t = A_t\alpha\left(\frac{n_t}{k_t}\right)^{1-\alpha}$$



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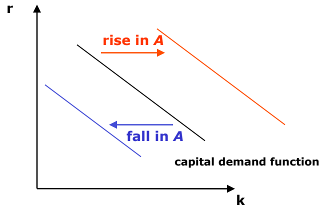

TFP SHOCKS AND CAPITAL/INVESTMENT DEMAND

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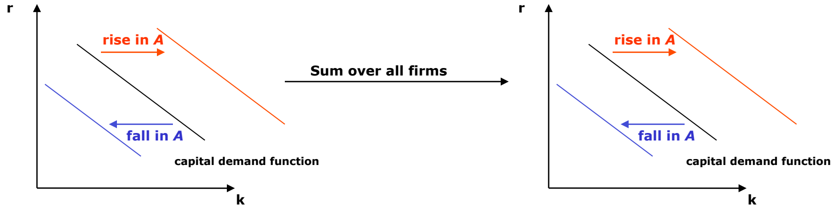

Firm-level capital demand function **Aggregate-level capital demand function**

- **IMPORTANT: TFP shocks shift the capital demand (and hence investment demand – recall $inv_t = k_{t+1} - k_t$) curve**

PREFERENCE SHOCKS

- **Illustrate idea using consumption-leisure framework**
  - **Preference shocks in consumption-savings framework: Problem Set 6**
- **Utility function (modified from Chapter 2): $u(Bc, l)$**
  - **c: consumption**
  - **l: leisure**
  - **B: preference shifter, with $B > 0$**
    - **Chapter 2: were implicitly considering $B = 1$**

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- **Mechanics of B**
  - **Makes *each* unit of c more (high B) desirable…**
  - **…or less (low B) desirable**
- **Interpretation of B**
  - **"Cultural" events that alter individuals' *desires***
  - **"Political" events that alter individuals' *desires***
  - **Any other events that alter individuals' *desires***

***Society-wide* events that alter a *given* person's desires – hence "taken as given" by an individual**

PREFERENCE SHOCKS

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- **MU of leisure same as always:** $\partial u / \partial l = u_2(Bc,l)$
- **→ MRS between consumption and leisure**
  - ***B* affects MRS in "two" ways – a "direct" effect and an "indirect" effect**

$$MRS_{c,l} = \frac{\partial u / \partial l}{\partial u / \partial c} = \frac{1}{B} \cdot \frac{u_2(Bc,l)}{u_1(Bc,l)}$$

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Focus on how the effects of *B* here alter indifference curves

("DIRECT EFFECT")

$$MRS_{c,l} = \frac{\partial u / \partial l}{\partial u / \partial c} = \frac{1}{B} \cdot \frac{u_2(Bc,l)}{u_1(Bc,l)}$$

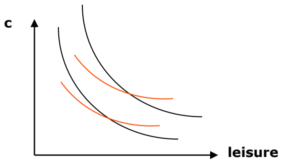
The effects of *B* here, to first-order, roughly cancel out (affects numerator and denominator in same way)

("INDIRECT EFFECT")

PREFERENCE SHOCKS AND INDIFFERENCE MAPS

$$MRS_{c,l} = \frac{\partial u / \partial l}{\partial u / \partial c} = \frac{1}{B} \cdot \frac{u_2(Bc,l)}{u_1(Bc,l)}$$

IF *B* RISES



Rise in *B* flattens all indifference curves (i.e., lowers *MRS* at any point in *c*-*l* space).

Interpretation: each unit of *c* more valuable, so *decreased willingness* to trade *c* for one more unit of *l*

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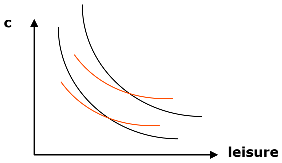

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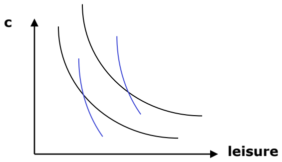
Superimpose a budget line: optimal choice of *c* and *l* clearly affected by shock to *B*

IF *B* FALLS



Fall in *B* steepens all indifference curves (i.e., raises *MRS* at any point in *c*-*l* space).

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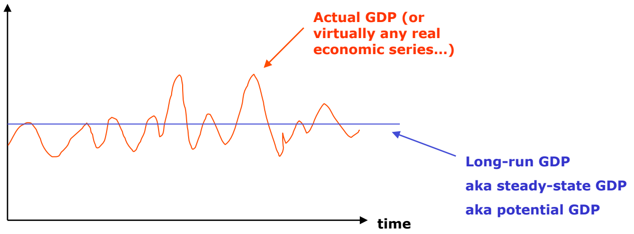
PREVIEW OF BUSINESS CYCLE THEORY

- ❑ **Modern macro view: periodic ups and downs of macroeconomic activity driven fundamentally by (various and many) shocks**



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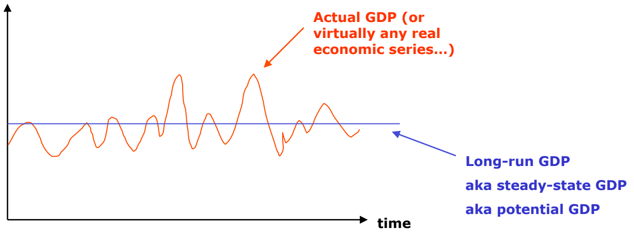

  - ❑ **Supply shocks: TFP shocks, others**
  - ❑ **Demand shocks: preference shocks, monetary policy shocks (Chapter 14), others**

- ❑ **Shocks over time lead to changes over time in**
  - ❑ **Consumers' incentives to work, save, and consume**
  - ❑ **Firms' incentives to hire, invest, and produce**

Economy's response(s) to shocks mediated through labor markets, capital markets, and goods markets

INTERTEMPORAL CONSUMPTION-LEISURE FRAMEWORK

MARCH 12, 2012

BASICS

- **Consumption-Leisure Framework**
  - **Foundation for goods-market demand and labor-market supply**
  - **Optimality condition**

$$\frac{\partial u / \partial l}{\partial u / \partial c} = (1-t)w$$

- **Consumption-Savings Framework**
  - **Foundation for (period-*t*) goods-market demand and asset-market supply**
  - **Optimality condition**

$$\frac{\partial u / \partial c_1}{\partial u / \partial c_2} = 1+r$$

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  - ☐ **Optimality condition**

$$\frac{\partial u/\partial c_1}{\partial u/\partial c_2} = 1+r$$

- ☐ **Bring together consumption-savings margin with the consumption-leisure margin**

Can put a β here

- ☐ **Utility function: $v(c_1, l_1, c_2, l_2) = u(c_1, l_1) + u(c_2, l_2)$**
  - ☐ **Dropping the assumption from simple (Chapter 3 and 4) two-period framework that income "falls from the sky"**
  - ☐ **Representative consumer has to work for his (labor) income in *each* period**

UTILITY AND BUDGET CONSTRAINTS

- ☐ **Utility function: $v(c_1, l_1, c_2, l_2) = u(c_1, l_1) + u(c_2, l_2)$**

- ☐ **Budget constraints**
  - ☐ **Period-1 budget constraint (nominal terms)**

$$P_1c_1 + A_1 - A_0 = iA_0 + (1-t_1)W_1(168-l_1)$$

  - ☐ **Period-2 budget constraint (nominal terms)**

$$P_2c_2 + A_2 - A_1 = iA_1 + (1-t_2)W_2(168-l_2)$$

Utility and Budget Constraints

- **Utility function: $v(c_1, l_1, c_2, l_2) = u(c_1, l_1) + u(c_2, l_2)$**

- **Budget constraints**
  - **Period-1 budget constraint (nominal terms)**

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  - **Period-2 budget constraint (nominal terms)**

$$P_2c_2 + A_2 - A_1 = iA_1 + (1-t_2)W_2(168-l_2)$$

- **Derive (nominal) LBC as usual (solve P2BC for A_1 and insert in P1BC)**

$$P_1c_1 + \frac{P_2c_2}{1+i} = (1-t_1)W_1(168-l_1) + \frac{(1-t_2)W_2(168-l_2)}{1+i} + (1+i)A_0$$

- **Or in real terms (work out details yourself)**

$$c_1 + \frac{c_2}{1+r} = (1-t_1)w_1(168-l_1) + \frac{(1-t_2)w_2(168-l_2)}{1+r} + (1+r)a_0$$

- **Or if infinite number of periods**

$$\sum_{t=0}^{\infty}\frac{c_t}{(1+r)^t} = \sum_{t=0}^{\infty}\frac{(1-t_t)w_t(168-l_t)}{(1+r)^t} + (1+r)a_0$$

  - **Assuming r is constant every period (slightly more complicated expression if r_t fluctuates every period)**

Consumption-Savings Margin

- **Describes decision of how much to consume in "short run" (period *t*) versus save for "long run" (period *t*+1 and beyond)**
  - **A decision that spans periods**

- **Think of as orthogonal to (i.e., independent of) the consumption-leisure margin**

- **Optimal choice (two-period framework) described by**

$$\frac{\partial u / \partial c_1}{\partial u / \partial c_2} = 1 + r$$

- **Optimal choice (infinite-period framework) described by**

$$\frac{\partial u / \partial c_t}{\partial u / \partial c_{t+1}} = 1 + r_t$$

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$$\frac{\partial u / \partial c_t}{\partial u / \partial c_{t+1}} = 1 + r_t, \quad \frac{\partial u / \partial c_{t+1}}{\partial u / \partial c_{t+2}} = 1 + r_{t+1}, \quad \frac{\partial u / \partial c_{t+2}}{\partial u / \partial c_{t+3}} = 1 + r_{t+2}, \quad \frac{\partial u / \partial c_{t+4}}{\partial u / \partial c_{t+5}} = 1 + r_{t+4},$$

etc.

  - **Recall: can think of infinite-period framework as sequence of overlapping two-period frameworks**

CONSUMPTION-LEISURE MARGIN

- **Describes decision within a period (i.e., focusing just on "short run") of how much to consume versus how much to work**
  - **A decision that does *not* span periods**
- **Think of as orthogonal to (i.e., independent of) the consumption-savings margin**
- **Optimal choice (two-period framework) described by**

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i.e., for *each* of the two periods

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- **Optimal choice (infinite-period framework) described by**

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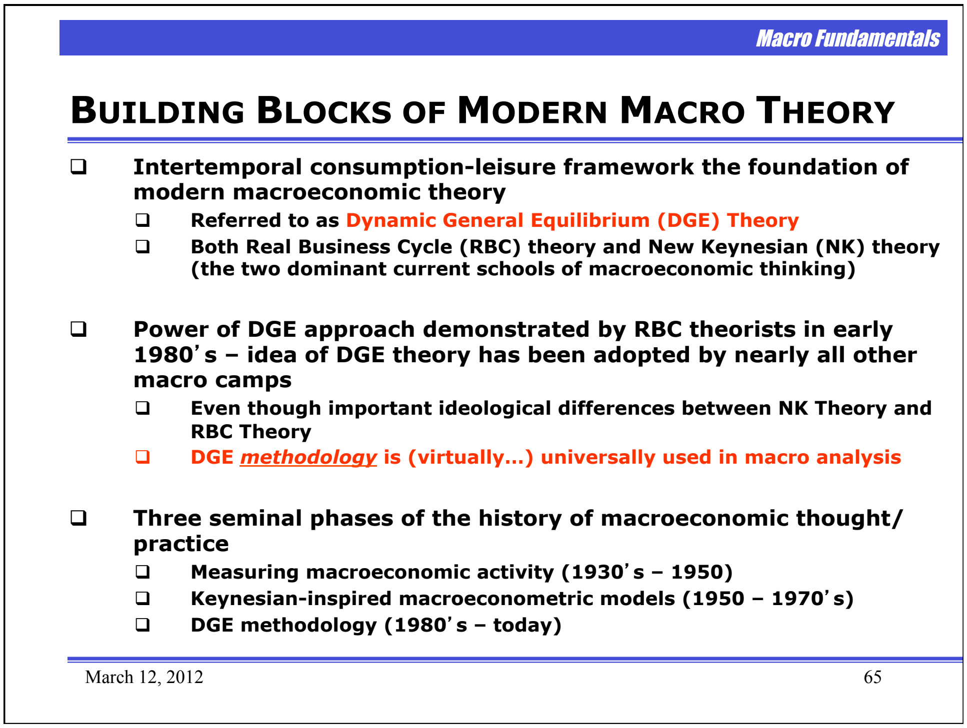
etc.

  - **Consumption-leisure decision "looks the same every period" in infinite-period environment**

BUILDING BLOCKS OF MODERN MACRO THEORY

- **Intertemporal consumption-leisure framework the foundation of modern macroeconomic theory**
  - **Referred to as Dynamic General Equilibrium (DGE) Theory**
  - **Both Real Business Cycle (RBC) theory and New Keynesian (NK) theory (the two dominant current schools of macroeconomic thinking)**
- **Power of DGE approach demonstrated by RBC theorists in early 1980's – idea of DGE theory has been adopted by nearly all other macro camps**
  - **Even though important ideological differences between NK Theory and RBC Theory**
  - **DGE *methodology* is (virtually…) universally used in macro analysis**



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- **Three seminal phases of the history of macroeconomic thought/ practice**
  - **Measuring macroeconomic activity (1930's – 1950)**
  - **Keynesian-inspired macroeconometric models (1950 – 1970's)**
  - **DGE methodology (1980's – today)**