Economics 602
Macroeconomic Theory and Policy
Problem Set 1
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1. **Partial Derivatives.** For each of the following multi-variable functions, compute the partial derivatives with respect to both *x* and *y*.
   a. $f(x,y) = xy$
   b. $f(x,y) = 2x + 3y$
   c. $f(x,y) = x^2 y^4$
   d. $f(x,y) = \ln x + 2\ln y$
   e. $f(x,y) = 2\sqrt{x} + 2\sqrt{y}$
   f. $f(x,y) = \frac{x}{y}$
   g. $f(x,y) = \frac{y}{x}$

2. **Properties of Indifference Maps.** For the general model of utility functions and indifference maps developed in class, explain why no two indifference curves can ever cross each other. Your answer must explain the economic logic here, and may also include appropriate equations and/or graphs.

3. **A Canonical Utility Function.** Consider the utility function

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

where c denotes consumption of some arbitrary good and σ (the Greek letter "sigma") is known as the "curvature parameter" because its value governs how curved the utility function is. In the following, restrict your attention to the region $c > 0$ (because "negative consumption" is an ill-defined concept). The parameter σ is treated as a constant.
   a. Plot the utility function for $\sigma = 0$. Does this utility function display diminishing marginal utility? Is marginal utility ever negative for this utility function?
   b. Plot the utility function for $\sigma = 1/2$. Does this utility function display diminishing marginal utility? Is marginal utility ever negative for this utility function?

c. Consider instead the natural-log utility function $u(c) = \ln(c)$. Does this utility function display diminishing marginal utility? Is marginal utility ever negative for this utility function?

d. Determine the value of σ (if any value exists at all) that makes the general utility function presented above collapse to the natural-log utility function in part c. (**Hint:** Examine the derivatives of the two functions.)

4. **The Implicit Function Theorem and the Marginal Rate of Substitution.** An important result from multivariable calculus is the **implicit function theorem,** which states that given a function $f(x, y)$, the derivative of y with respect to x is given by

$$\frac{dy}{dx} = -\frac{\partial f / \partial x}{\partial f / \partial y},$$

where $\partial f / \partial x$ denotes the partial derivative of f with respect to x and $\partial f / \partial y$ denotes the partial derivative of f with respect to y. Simply stated, a partial derivative of a multivariable function is the derivative of that function with respect to one particular variable, treating all other variables as constant. For example, suppose $f(x, y) = xy^2$. To compute the partial derivative of f with respect to x, we treat y as a constant, in which case we obtain $\partial f / \partial x = y^2$, and to compute the partial derivative of f with respect to y, we treat x as a constant, in which case we obtain $\partial f / \partial y = 2xy$.

We have described the slope of an indifference curve as the marginal rate of substitution between the two goods. Imagining that c_2 is plotted on the vertical axis and c_1 plotted on the horizontal axis, compute the marginal rate of substitution for the following utility functions.

a. $u(c_1, c_2) = \ln(c_1) + \ln(c_2)$

b. $u(c_1, c_2) = \sqrt{c_1} + \sqrt{c_2}$

c. $u(c_1, c_2) = c_1^a c_2^{1-a}$, where $a \in (0,1)$ is some constant.

5. **Sales Tax.** Consider the standard consumer problem we have been studying, in which a consumer has to choose consumption of two goods c_1 and c_2 which have prices (in terms of money) P_1 and P_2, respectively. These prices are prices before any applicable taxes. Many states charge sales tax on some goods but not on others – for example, many states charge sales tax on all goods except food and clothes. Suppose that good 1 carries a per-unit sales tax, while good 2 has no sales tax. Use the variable t_1 to denote this sales tax, where t_1 is a number between zero and one (so, for example, if the sales tax on good 1 were 15 percent, we would have $t_1 = 0.15$).

a. With sales tax t_1 and consumer income Y, write down the budget constraint of the consumer. Explain economically how/why this budget constraint differs from the standard one we have been considering thus far.

b. Graphically describe how the imposition of the sales tax on good 1 alters the optimal consumption choice (ie, how the optimal choice of each good is affected by a policy shift from $t_1 = 0$ to $t_1 > 0$).

c. Suppose the consumer's utility function is given by $u(c_1, c_2) = \log c_1 + \log c_2$. Using a Lagrangian, solve algebraically for the consumer's optimal choice of c_1 and c_2 as functions of P_1, P_2, t_1, and Y. Graphically show how, for this particular utility function, the optimal choice changes due to the imposition of the sales tax on good 1.