Economics 602
Macroeconomic Theory and Policy
Problem Set 2
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1. **Interaction of Consumption Tax and Wage Tax.** A basic idea of President Bush's economic advisers throughout his administration was to try to move the U.S. further away from a system of investment taxes (which we will discuss later in the course) and more towards a system of consumption taxes**.** A nationwide consumption tax would essentially be a national sales tax. Here, you will modify our basic consumption-leisure model to include both a proportional wage tax (which we will now denote by t_n, where, as before, $0 \leq t_n < 1$) as well as a proportional consumption tax (which we will denote by t_c, where $0 \leq t_c < 1$). A proportional consumption tax means that for every dollar on the price tags of items the consumer buys, the consumer must pay $(1+t_c)$ dollars. Throughout the following, suppose that economic policy has no effect on wages or prices (that is, the nominal wage W and the price of consumption P are constant throughout).

    a. Construct the budget constraint in this modified version of the consumption-leisure model. Briefly explain economically how this budget constraint differs from that in the standard consumption-leisure model we have studied in class.
    b. Suppose currently the federal wage tax rate is 20 percent ($t_n = 0.20$) while the federal consumption tax rate is 0 percent ($t_c = 0$), and that the Bush economic team is considering proposing lowering the wage tax rate to 15 percent. However, they wish to leave the representative agent's optimal choice of consumption and leisure unaffected. Can they simultaneously increase the consumption tax rate from its current zero percent to achieve this goal? If so, compute the new associated consumption tax rate, and explain the economic intuition. If not, explain mathematically as well as economically why not.
    c. A **tax policy** is defined as a particular combination of tax rates. For example a labor tax rate of 20 percent combined with a consumption tax rate of zero percent is one particular tax policy. A labor tax rate of five percent combined with a consumption tax rate of 10 percent is a different tax policy. Based on what you found in parts a and b above, address the following statement: a government can use many different tax policies to induce the same level of consumption by individuals.
    d. Consider again the Bush proposal to lower the wage tax rate from 20 percent to 15 percent. This time, however, policy discussion is focused on trying to boost overall consumption. Is it possible for this goal to be achieved if the consumption tax rate is raised from its current zero percent?
    e. Using a Lagrangian, derive the consumer's consumption-leisure optimality condition (for an arbitrary utility function) as a function of the real wage and the consumption and labor tax rates.

2. **Non-Backward-Bending Labor Supply Curve.** Consider an economy populated by 100 individuals who have identical preferences over consumption and leisure. In this economy, the aggregate labor supply curve is upward-sloping. For simplicity, suppose throughout this question that the labor tax rate is zero.

    a. For such a labor supply curve, how does the substitution effect compare with the income effect?
    b. Using indifference curves and budget constraints, show how such a labor supply curve arises.

3. **A Backward-Bending Aggregate Labor Supply Curve?** Despite our use of the backward-bending labor supply curve as arising from the representative agent's preferences, there is controversy in macroeconomics about whether this is a good representation. Specifically, even though a backward-bending labor supply curve may be a good description of a given individual's decisions, it does **not** immediately follow that the representative agent's preferences should also feature a backward-bending labor supply curve. In this exercise you will uncover for yourself this problem. For simplicity, assume that the labor tax rate is $t = 0$ throughout all that follows.

    a. Suppose the economy is made up of five individuals, person A, person B, person C, person D, and person E, each of whom has the labor supply schedule given below. Using the indicated wage rates, graph each individual's labor supply curve **as well as** the aggregate labor supply curve.

Nominal Wage, W	Person A	Person B	Person C	Person D	Person E
\$10	20 hours	0 hours	0 hours	0 hours	0 hours
\$15	25	15	0	0	0
\$20	30	22	8	0	0
\$25	33	27	15	5	0
\$30	35	30	20	15	0
\$35	37	32	25	20	6
\$40	36	31	27	25	21
\$45	35	30	26	28	30
\$50	33	29	24	25	29

    Now suppose that in this economy, the "usual" range of the nominal wage is between \$10 and \$45.

    b. Restricting attention to this range, is the aggregate labor supply curve backward-bending?
    c. At a theoretical level, if we want to use the representative-agent paradigm and restrict attention to this usual range of the wage, does a backward-bending labor supply curve make sense?

d. Explain qualitatively the relationship you find between the individuals' labor supply curves and the aggregate labor supply curve over the range \$10 – \$45. Especially address the "backward-bending" nature of the curves.

4. **Consumption, Labor, and Unemployment: Fiscal Policy Choices in a Search Framework.** The 2010 Nobel Prize in Economics was awarded to Peter Diamond, Dale Mortensen, and Christopher Pissarides for their development (during the 1970s and 1980s) of search theory. Search theory is a framework especially suited for studying labor market issues. The search framework builds on, but is richer than, the basic theory of supply and demand. Search theory can be applied to both the supply side of the labor market (building on the analysis of Chapter 2) as well as the demand side of the labor market (building on the analysis of Chapter 6, which we will study later in the course). In what follows, you will study the application of search theory to the supply side of the labor market.

There are three basic ideas underlying search theory. First, search theory incorporates into basic supply-and-demand analysis the fact that when an individual wants to work (i.e., "supplies labor"), there is a chance that employment may not be found. That is, an individual "searching" for a job has a **probability less than one** that a suitable "match" will be found.

Second, as a direct consequence of the probabilistic nature of successfully finding a job, there is a **probability larger than zero** that an individual might end up "unemployed" – that is, having searched for work but not found anything. In this case, he/she receives "unemployment benefits" from the government.

Third, search theory makes explicit the **costs associated with search activity.** As is realistic, when an individual wants a job, he/she does not simply "go to the market" as in basic supply-and-demand analysis. Rather, the individual must expend resources **searching** for a job (think of these costs as due to the time spent looking at recruiting advertisements through various web and networking channels, at career fairs, going through the interviewing process, etc.).

We will incorporate these three ideas into the **one-period consumption-labor framework** of Chapter 2, thereby enriching the range of predictions that it can generate and policy advice it may be able to offer. To do so, first introduce some notation:

p^{FIND}: the **probability** that an individual searching for a job finds suitable employment. By the definitions of probabilities, $p^{FIND} \in [0,1]$ (that is, the probability is a number between zero and one). **Hence, the probability of not finding a job is 1-p^{FIND}.** (**Note:** p does **not** denote a "price.")

s: the "search cost," measured in real units (that is, in units of consumption goods) that an individual incurs for **each hour that he/she would like to work.** For example, if the individual desires n =10 hours of work during the week, the total search cost is 10s; if the individual desires n = 20 hours of work during the week, the total search cost is 20s; and so on. The way to interpret this is that it is more costly (in a search sense) to find a job the closer it is to a "full time" job because one has to send out more applications, go through more interviews, etc. The search cost is $s \geq 0$.

b: the "unemployment benefit," measured in real units (that is, in units of consumption goods) that an individual receives for **each hour that he/she does not work**. For example, if the individual does **not** work (which is tantamount to "taking leisure") for l = 50 hours during the week, he/she receives a total of 50b in unemployment benefits; if the individual does **not** work

for l = 100 hours during the week, he/she receives a total of 100b in unemployment benefits; and so on. **In principle, the unemployment benefit is $b \geq 0$. However, we will focus on the case in which b = 0 exactly, even though the term b does appear in the expressions below.**

In quantitative and policy applications that use this framework, a commonly-used utility function is

$$u(c,l) = \ln c - \frac{\theta}{1+1/\psi}(168-l)^{1+1/\psi},$$

in which ψ and θ (the Greek letters "psi" and "theta," respectively) are **constants** (even though we will not assign any numerical value to them) in the utility function. The representative individual has no control over either ψ or θ, and both $\psi > 0$ and $\theta > 0$. You are to use this utility function throughout the analysis.

The budget constraint, expressed in **real units** (that is, in units of consumption goods), is

$$c + sn = p^{FIND}(1-t)wn + \left(1-p^{FIND}\right)bl,$$

in which, w denotes the **real wage** and t the labor income tax rate. The right hand side (the income side) of the budget constraint is expressed in "**expected value form"** because of the fact that two **mutually exclusive** things can occur: a job is found (which occurs with probability p^{FIND}), in which case income is after-tax wage earnings; or a job is not found (which occurs with probability 1- p^{FIND}), in which case income is the total unemployment benefits received from the government.[1] **AGAIN, note that we will consider only the case of b = 0 exactly, even though it appears in the expression above.**

To complete the description of the (representative) individual's utility maximization problem:

- Just as in Chapter 2, adopt a weekly view, so that $n + l = 168$, with n denoting the number of hours that an individual works, and l the number of hours spent not working.
- The variables **taken as given** by the individual are real wages, the probability of finding a suitable job, the search cost per hour of (desired) work, and the unemployment benefit per hour of non-work. That is, the individual takes $\left(w, p^{FIND}, t, s, b\right)$ as given when solving his/her utility maximization problem.

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[1] The "expected value" form of the budget constraint arises from application of the probability and statistics concept of "expectations" of uncertain events (here, "getting a job" is an uncertain event). For our purposes, you can simply take the budget constraint as written as given, with no need to connect it to the underlying probability and statistics framework.

Problem 4 continued

a. Using the setup of the problem, algebraically re-arrange the given budget constraint so that, in your final expression, the variables c and l each appear **only** on the left hand side, and the variable n does **not** directly appear at all. Clearly present the steps and logic of your work. **(Note:** the correct expression for the budget constraint is critical for all of the analysis that follows, so you should make sure that your work here is absolutely correct! If the budget constraint here is incorrect, we will **not** necessarily "carry through the error" all the way through the remainder of your analysis when reviewing solutions.)

b. Based on the budget constraint in part a, construct the Lagrangian for the consumer's utility maximization problem. Clearly present the steps and logic of your analysis.

c. Based on the Lagrangian constructed in part b, compute the first-order conditions with respect to both c and l. **(Note: your analysis is to be based on the utility function given above).** Clearly present the steps and logic of your analysis.

d. Based on the two first-order conditions computed in part c, construct the consumption-leisure optimality condition. The final expression must read

$$\frac{u_l(c,l)}{u_c(c,l)} = \ldots$$

in which the right hand side of the expression is for you to determine. **Your final expression may NOT include any Lagrange multipliers in it.** You should present very clearly the algebraic steps involved in constructing this expression.

e. Qualitatively sketch the consumption-leisure optimality condition obtained in part d in a graph with c on the vertical axis and l on the horizontal axis. **Clearly label the slope of the budget line in the sketch.**

Problem 4 continued

Due to the economic downturn and associated sluggishness in employment, the government has been considering (and engaging in) various forms of interventions in labor markets aimed at increasing the welfare (the utility) of individuals. Based on the sketch in part e, you are to analyze various types of labor market interventions with a focus on determining whether or not they would increase the welfare (the utility) of the representative individual. **(Note:** you are not required to draw new sketches in the subsequent analysis, but you may do so if it clarifies your work.)

f. Based on and referring to the sketch in part e, would a reduction in the labor income tax rate t increase utility, decrease utility, or leave utility unchanged? Or is it impossible to determine? **Clearly and briefly** describe the **economic interpretation** (that is, not simply a verbal re-statement of the mathematical or graphical analysis) for your conclusion.

g. Based on and referring to the sketch in part e, would an increase in the unemployment benefit b increase utility, decrease utility, or leave utility unchanged? Or is it impossible to determine? **Clearly and briefly** describe the **economic interpretation** (that is, not simply a verbal re-statement of the mathematical or graphical analysis) for your conclusion. **(NOTE: in our analysis, we can skip this question.)**

h. Based on and referring to the sketch in part e, would policies aimed at reducing the search cost s incurred by individuals increase utility, decrease utility, or leave utility unchanged? Or is it impossible to determine? **Clearly and briefly** describe the **economic interpretation** (that is, not simply a verbal re-statement of the mathematical or graphical analysis) for your conclusion.

i. Based on and referring to the sketch in part e, would policies aimed at increasing the probability p^{FIND} that individuals can find suitable jobs increase utility, decrease utility, or leave utility unchanged? Or is it impossible to determine? **Clearly and briefly** describe the **economic interpretation** (that is, not simply a verbal re-statement of the mathematical or graphical analysis) for your conclusion.