Department of Applied Economics Johns Hopkins University

Economics 602
Macroeconomic Theory and Policy
Problem Set 4
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1. **Optimal Choice in the Consumption-Savings Model with Credit Constraints: A Numerical Analysis.** Consider our usual two-period consumption-savings model. Let preferences of the representative consumer be described by the utility function

$$u(c_1, c_2) = \sqrt{c_1} + \beta\sqrt{c_2},$$

where c_1 denotes consumption in period one and c_2 denotes consumption in period two. The parameter β is known as the subjective discount factor and measures the consumer's degree of impatience in the sense that the smaller is β, the higher the weight the consumer assigns to present consumption relative to future consumption. Assume that $\beta = 1/1.1$. For this particular utility specification, the marginal utility functions are given by $u_1(c_1, c_2) = \dfrac{1}{2\sqrt{c_1}}$ and $u_2(c_1, c_2) = \dfrac{\beta}{2\sqrt{c_2}}$.

The representative household has initial **real** financial wealth (including interest) of $a_0 = 1$. The household earns $y_1 = 5$ units of goods in period one and $y_2 = 10$ units in period two. The real interest rate paid on assets held from period one to period two equals 10% (i.e., $r_1 = 0.1$).

a. Calculate the equilibrium levels of consumption in periods one and two (**Hint:** Set up the Lagrangian and solve.)
b. Suppose now that lenders to this consumer impose **credit constraints** on the consumer. Specifically, they impose the tightest possible credit constraint – the consumer is not allowed to be in debt at the end of period one, which implies that the consumer's real wealth at the end of period one must be nonnegative ($a_1 \geq 0$) (**Note:** here, a_1 is defined as being exclusive of interest, in contrast to the definition of a_0 above). What is the consumer's choice of period-one and period-two consumption under this credit constraint? Briefly explain, either logically or graphically or both.
c. Does the credit constraint described in part b enhance or diminish welfare (i.e., does it increase or decrease lifetime utility)? Specifically, find the level of lifetime utility under the credit constraint and compare it to the level of lifetime utility under no credit constraint.

Suppose now that the consumer experiences a temporary increase in real income in period one to $y_1 = 9$, with real income in period two unchanged.

d. Calculate the effect of this positive surprise in income on c_1 and c_2, supposing that there is no credit constraint on the consumer.
e. Finally, suppose that the credit constraint described in part b is back in place. Will it be binding? That is, will it affect the consumer's choices?

2. **Government in the Two-Period Economy.** Consider again our usual two-period consumption-savings model, augmented with a government sector. Each consumer has preferences described by the utility function

$$u(c_1, c_2) = \ln c_1 + \ln c_2,$$

where ln stands for the natural logarithm, c_1 is consumption in period one, and c_2 is consumption in period two.

Suppose that both households and the government start with zero initial assets (i.e., $A_0 = 0$ and $b_0 = 0$), and that the real interest rate is always 10 percent. Assume that government purchases in the first period are one ($g_1 = 1$) and in the second period are 9.9 ($g_2 = 9.9$). In the first period, the government levies lump-sum taxes in the amount of 8 ($t_1 = 8$). Finally, the real incomes of the consumer in the two periods are $y_1 = 9$ and $y_2 = 23.1$.

a. What are lump-sum taxes in period two (t_2), given the above information?
b. Compute the optimal level of consumption in periods one and two, as well as nationals savings in period one.
c. Consider a tax cut in the first period of 1 unit, with government purchases left unchanged. What is the change in national savings in period one? Provide intuition for the result you obtain.
d. Now suppose again that $t_1 = 8$ and also that credit constraints on the consumer, of the type described in Question 1, are in place, with lenders stipulating that consumers cannot be in debt at the end of period one (i.e., the credit constraint again takes the form $a_1 \geq 0$). Will this credit constraint affect consumers' optimal decisions? Explain why or why not. Is this credit constraint welfare-enhancing, welfare-diminishing, or welfare-neutral?
e. Now with the credit constraint described above in place, consider again the tax cut of 1 unit in the first period, with no change in government purchases. (That is, t_1 falls from 8 units to 7 units.) What is the change in national savings in period one that arises due to the tax cut? Provide economic intuition for the result you obtain.

3. "Marginal Propensity to Consume" for Various Utility Functions. An old (Keynesian) idea in macroeconomics is the "marginal propensity to consume," abbreviated MPC. Briefly, the MPC is the fraction of current-period income that is consumed in the current period. For example, if income in period one is 2 and consumption in period one is 1.8, the $MPC = 0.9$.

Consider the standard two-period consumption-savings model in which the representative consumer has no control over his real labor income in periods 1 and 2, denoted by y_1 and y_2, respectively. As usual, denote by r the real interest rate between period 1 and period 2, and assume the individual begins his life with zero initial assets ($a_0 = 0$). Make the additional assumption that in present-discounted-value terms, his real income in each of the two periods is the same – that is, $y_1 = \frac{y_2}{1+r}$. Using the LBC and the intertemporal optimality condition, derive for each of the following utility functions the "period-1 MPC" – that is, derive what fraction of period-1 real income the consumer devotes to period-1 consumption. (In other words, derive the coefficient MPC in the expression $c_1 = MPC \cdot y_1 + const$.) Note that not all of these utility functions satisfy the property that utility is strictly concave in both its arguments – but this is irrelevant for the exercise here. **(Hint:** Set up the Lagrangian in order to solve.)

a. $u(c_1, c_2) = \sqrt{c_1} + \sqrt{c_2}$.

b. $u(c_1, c_2) = \ln(c_1) + c_2$ (No, this is not a typo…).

c. $u(c_1, c_2) = c_1^{b} c_2^{1-b}$**,** where b is a constant such that $0 < b < 1$. (This type of utility function is called the Cobb-Douglas utility function).