Economics 602
Macroeconomic Theory and Policy
Problem Set 6
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1. **Lags in Labor Hiring.** Rather than supposing that the representative firm at the beginning of period t can decide how much labor it would like to hire for use in period t, suppose that labor used in period t must be chosen in period t-1. (That is, suppose n is a stock (aka state) variable.) As usual, capital for use in production in period t must be purchased in period t-1 because of the "time to build" surrounding capital goods. With this lag in labor hiring, construct the lifetime (in the two-period model) profit function of the firm, and show that the real interest rate now is a relevant price for labor as well as capital goods. Provide brief economic intuition. (**Hint:** Make as close an analogy with our model of firm ownership of capital as you can – in particular, think of workers in this model as being "owned" (contractually obligated to) firms.)

2. **Preference Shocks in the Consumption-Savings Model.** In the two-period consumption-savings model (in which the representative consumer has no control over his real labor income y_1 and y_2), suppose the representative consumer's utility function is $u(c_1, Bc_2)$, where, as usual, c_1 denotes consumption in period 1, c_2 denotes consumption in period 2, and B is a preference parameter.

    a. Use an indifference-curve/budget-constraint diagram to illustrate the effect of an increase in B on the consumer's optimal choice of period-1 consumption.
    b. Illustrate the effect of an increase in B on the private savings function. Provide economic interpretation for the result you find.
    c. In the months preceding the U.S. invasion of Iraq, data shows that consumers decreased their consumption and increased their savings. Is an increase in B and the effects you analyzed in parts a and b above consistent with the idea that consumption fell and savings increased because of a looming war? If so, explain why; if not, explain why not.
    d. Using a Lagrangian and assuming the utility function is $u(c_1, B \cdot c_2) = \ln(c_1) + \ln(B \cdot c_2)$, show how the representative consumer's MRS depends on B.
    e. How would your analysis in parts a and b change if the consumer's utility function were $u(Dc_1, c_2)$ (instead of $u(c_1, Bc_2)$) and you were told that the value D decreased? (D is simply some other measure of preference shocks.)

3. **Intertemporal Consumption-Leisure Model – A Numerical Look.** Consider the intertemporal consumption-savings model. Suppose the lifetime utility function is given by $v(B_1c_1, l_1, B_2c_2, l_2) = u(B_1c_1, l_1) + u(B_2c_2, l_2)$, which is a slight modification of the utility function presented in Chapter 5. The modification is that preference shifters B_1 and B_2 enter the lifetime utility function, with B_1 the preference shifter in period one and B_2 is the preference shifter in period two. In each of the two periods the function u takes the form

$$u(B_tc_t, l_t) = 2\sqrt{B_tc_t} + 2\sqrt{l_t}\ .$$

Note the t subscripts -- $t = 1,2$ depending on which period we are considering. Labor tax rates, real wages, the real interest rate between period one and period two, and the preference realizations are given by: $t_1 = 0.15$, $t_2 = 0.2$, $w_1 = 0.2$, $w_2 = 0.25$, $r = 0.15$, $B_1 = 1$, $B_2 = 1.2$. Finally, the initial assets of the consumer are zero.

a. Construct the marginal rate of substitution functions between consumption and leisure in each of period one and period two (**Hint:** these expressions will be functions of consumption and leisure – you are not being asked to solve for any numerical values yet). How does the preference shifter affect this intratemporal margin?
b. Construct the marginal rate of substitution function between period-one consumption and period-two consumption. (**Hint:** Again, you are not being asked to solve for any numerical values yet.) How do the preference shifters affect this intertemporal margin?
c. Using the expressions you developed in parts a and b along with the lifetime budget constraint (expressed in **real** terms…) and the given numerical values, solve numerically for the optimal choices of consumption in each of the two periods and of leisure in the two periods. (**Hint:** You need to set up and solve the appropriate Lagrangian.) (**Note:** the computations here are messy and the final answers do not necessarily work out "nicely." **To preserve some numerical accuracy, carry out your computations to at least four decimal places.**)
d. Based on your answer in part c, how much (in real terms) does the consumer save in period one? What is the asset position that the consumer begins period two with?
e. Suppose B_2 were instead higher, at 1.6. How are your solutions in parts c and d affected? Provide brief interpretation in terms of "consumer confidence."

4. **Search Theory and Labor Demand.** The 2010 Nobel Prize in Economics was awarded to Peter Diamond, Dale Mortensen, and Christopher Pissarides for their development (during the 1970s and 1980s) of search theory. Search theory is a framework especially suited for studying labor market issues. The search framework builds on, but is richer than, the basic theory of supply and demand. Search theory can be applied to both the supply side of the labor market (building on the analysis of Chapter 2) as well as the demand side of the labor market (building on the analysis of Chapter 6). In what follows, you will study the application of search theory to the demand side of the labor market.

There are three basic ideas underlying search theory. First, search theory incorporates into basic supply-and-demand analysis the fact that when a firm wants to hire a worker (i.e., "demands labor"), there is a chance that a suitable worker may not be found. That is, a firm "searching" for a worker has a **probability less than one** that a suitable "match" will be found.

Second, search theory makes explicit the **costs associated with search activity.** As is realistic, when a firm wants to hire a worker, it does not simply "go to the market" as in basic supply-and-demand analysis. Rather, it must expend resources **searching** for a worker (think of these costs as the recruiting costs inherent in running a firm's human resources department, placing job advertisements in various outlets, the interviewing process, etc.). Moreover, because of the various activities involved in the search, or "recruiting," process, there is a time delay between when a firm engages in recruiting activities and when, if recruiting is successful, a new employee actually begins working at the firm. For concreteness, suppose that if a firm successfully recruited a new employee in period t, it is not until period $t+1$ that the new employee actually begins working.

Third, due to recruiting costs and time delays in the recruiting process, if a firm successfully hires a new employee, that employee will (typically) work for the firm for more than just one period. That is, in labor markets, **multi-period relationships between workers and firms** are the norm rather than the exception. (For example, in the U.S., the average length of time a worker remains in a particular job is between two and three years.)

To formalize these three ideas in the context of an **infinite-period firm profit-maximization problem,** introduce some notation:

Problem 4 continued

p_t^{FIND}: the **probability**, in period *t*, that a firm searching to fill a particular job opening finds a suitable worker. By the definitions of probabilities, $p_t^{FIND} \in [0,1]$. (Define in an analogous way p_{t+1}^{FIND}, p_{t+2}^{FIND}, p_{t+3}^{FIND}, and so on.)

J_t: the "recruiting cost," in period *t*, measured in real terms, that is associated with **each** job opening that a firm is trying to fill; the recruiting cost is $J_t \geq 0$. (Define in an analogous way J_{t+1}, J_{t+2}, J_{t+3}, and so on.)

$p_t^{TURNOVER}$: the **probability** that a worker **employed in a particular job in period *t*** will **NOT** be employed at that same job in period *t*+1 (whether due to quitting or being fired, each of which is a form of worker "turnover"). By the definitions of probabilities, $p_t^{TURNOVER} \in [0,1]$. (Define in an analogous way $p_{t+1}^{TURNOVER}$, $p_{t+2}^{TURNOVER}$, $p_{t+3}^{TURNOVER}$, and so on.)

v_t: the number of job **vacancies** in period *t* that a firm is attempting to fill (that is, the number of job openings the firm has and is actively recruiting for). The cost of "setting up" each vacancy (the administrative cost associated with recruiting) is the cost J_t described above. (Define in an analogous way v_{t+1}, v_{t+2}, v_{t+3}, and so on.)

Supposing that the (representative) firm has "many" employees, the way in which the total number of employees that it has on its payroll changes from period *t* to period *t*+1 is

$$n_{t+1} = \left(1 - p_t^{TURNOVER}\right) n_t + v_t p_t^{FIND},$$

which is to be understood as follows: the number of employees that work at the firm in period *t* is n_t, and then, because some new workers are hired in period *t* and some existing workers turn over between period *t* and *t*+1, the firm has a (possibly different) number of employees in period *t*+1, n_{t+1}. Similarly, the way in which the total number of employees that the firm has on its payroll changes from period *t*+1 to period *t*+2 is

$$n_{t+2} = \left(1 - p_{t+1}^{TURNOVER}\right) n_{t+1} + v_{t+1} p_{t+1}^{FIND},$$

the way in which the total number of employees that the firm has on its payroll changes from period *t*+2 to period *t*+3 is

$$n_{t+3} = \left(1 - p_{t+2}^{TURNOVER}\right) n_{t+2} + v_t p_{t+2}^{FIND},$$

Problem 4 continued

and so on. Refer to these previous three expressions (along with their analogs in periods *t*+3, *t*+4, etc) as the firm's **employment constraints.**

To complete the description of the (representative) firm's (dynamic) profit-maximization problem starting from the beginning of period *t*:

- In period *t*, total output and hence total revenue of the firm (denominated in real terms) is $A_t f(k_t, n_t)$, in which A_t denotes total factor productivity (TFP) in period *t*, k_t is the amount of capital ("machines and equipment") the firm has at the start of period *t* (recall from Chapter 6 that capital is a stock variable that "takes one period to build"), and $f(k_t, n_t)$ is the firm's production function. Similarly, for period *t*+1, total output and hence total revenue of the firm (denominated in real terms) is $A_{t+1} f(k_{t+1}, n_{t+1})$, and so on.
- The real interest rate between any two consecutive time periods is **always *r* > 0** (that is, the real interest does not change over time, which is indicated by the lack of a time subscript).
- The real wage the firm must pay each worker in period *t* is w_t, which is taken as given by the firm. Similarly, the real wage the firm must pay each worker in period *t*+1 is w_{t+1}, in period *t*+2 is w_{t+2}, and so on.
- The variables **taken as given** by the firm are real wages, the probabilities of finding workers, and the probabilities of worker turnover. That is, the firm takes $\left(w_t, p_t^{FIND}, p_t^{TURNOVER}\right)$ as given in period *t*, takes $\left(w_{t+1}, p_{t+1}^{FIND}, p_{t+1}^{TURNOVER}\right)$ as given in period *t*+1, takes $\left(w_{t+2}, p_{t+2}^{FIND}, p_{t+2}^{TURNOVER}\right)$ as given in period *t*+2, and so on.
- In period *t*, the firm's profit function (in real terms) is

$$A_t f(k_t, n_t) + k_t - w_t n_t - k_{t+1} - J_t v_t,$$

  which, except for the inclusion of "total recruiting costs" $J_t v_t$, is identical to the analysis in Chapter 6. Thus, the firm's profit function (in real terms) in period *t*+1 is $A_{t+1} f(k_{t+1}, n_{t+1}) + k_{t+1} - w_{t+1} n_{t+1} - k_{t+2} - J_{t+1} v_{t+1}$, and so on for periods *t*+2, *t*+3, etc.

With this background in place, your analysis is to proceed as follows.

Problem 4 continued

a. Construct an infinite-period Lagrangian (starting from the beginning of period *t*) for the representative firm's (infinite-period) profit-maximization problem. **This Lagrangian must take into account the employment constraints described above,** along with correctly incorporating all of the other pieces of the theory described above. Use λ_t as your notation for the Lagrange multiplier on the period-*t* employment constraint, λ_{t+1} as the Lagrange multiplier on the period-(*t*+1) employment constraint, and so on. **Because the Lagrangian has an infinite number of terms, write out the first several terms to make clear the nature of the Lagrangian, and provide any explanation needed in constructing the Lagrangian. (Note 1:** use the two-period analysis of firm theory in Chapter 6 as your intuitive basis for constructing the Lagrangian.) **(Note 2:** the Lagrangian is critical for all of the analysis that follows, so you should make sure that your work here is absolutely correct!)

b. Based on the Lagrangian in part a, compute the first-order condition with respect to k_{t+1} (that is, with respect to how much capital the firm would optimally like to use in its production process in period *t*+1).

c. Based on the first-order condition computed in part b, explain (using any appropriate combination of mathematical analysis, graphical analysis, and logic) how the "search" aspects of labor markets affect the firm's capital demand decisions. **For this part of the problem, you may (but do not need to) suppose that the production function is Cobb-Douglas:** $f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}$, with $\alpha \in (0,1)$.

d. Based on the Lagrangian in part a, compute the following three first-order conditions: with respect to v_t, with respect to n_{t+1}, and with respect to v_{t+1} (that is, with respect to how many job openings ("vacancies") the firm optimally chooses in period *t* and period *t*+1, and how many employees the firm would optimally like to have on its payroll at the beginning of period *t*+1).

e. Based on the three first-order conditions computed in part d, construct an expression that reads

$$\frac{J_t}{p_t^{FIND}} = \dots,$$

in which the right-hand-side of the expression is for you to determine. **Your final expression may NOT include any Lagrange multipliers in it. (**You should make very clear the algebraic steps involved in constructing this expression.)

Problem 4 continued

The expression obtained in part e is known as the **job creation condition,** which is the central analytical prediction of search theory. In the remainder of the analysis, you will compare and contrast the job creation condition with the "labor demand" condition studied in Chapter 6. For the remainder of the analysis, **you may (but do not need to) suppose that the production function is Cobb-Douglas:** $f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}$, with $\alpha \in (0,1)$.

f. Consider the job creation condition in part e. Suppose that all workers turn over every period – that is, suppose

$$p_t^{TURNOVER} = p_{t+1}^{TURNOVER} = p_{t+2}^{TURNOVER} = p_{t+3}^{TURNOVER} = ...1.$$

With this assumption, what does the job creation condition simplify to? Briefly, but carefully, describe the economic interpretation of the job creation condition in this case?

g. Consider the job creation condition in part e. As in part f, suppose that all workers turn over every period – that is, suppose

$$p_t^{TURNOVER} = p_{t+1}^{TURNOVER} = p_{t+2}^{TURNOVER} = p_{t+3}^{TURNOVER} = ...1.$$

In addition, suppose that a firm can **always** find a suitable worker – that is, suppose

$$p_t^{FIND} = p_{t+1}^{FIND} = p_{t+2}^{FIND} = p_{t+3}^{FIND} = ...1.$$

With these assumptions, what does the job creation condition simplify to? Briefly, but carefully, describe the economic interpretation of the job creation condition in this case?

h. Analytically, can the job creation condition in part e be simplified so that it becomes **identical** to the labor demand condition studied in Chapter 6? **If so, describe the entire set of assumptions needed to make the two identical** (these assumptions would be of the form "variable *x* must have the numerical value *y*"). **If not, describe why there is no set of assumptions that makes the job creation condition identical to the labor demand condition. In either case, briefly and qualitatively describe the economics of why (or why not) the two conditions can be made to coincide. (Hint:** your analysis here may build on your analysis in part f and/or part g; if you do so, carefully explain how your analysis builds on part f and/or part g.)

Problem 4 continued

Consider again the job creation condition in part e, now ignoring all of the assumptions, analysis, and conclusions of part f, part g, and part h. S**uppose that the job creation condition reaches a steady state, in which all real variables *and all probabilities* (namely, p_t^{FIND} and $p_t^{TURNOVER}$) stop fluctuating from one time period to the next.** Construct **one single diagram** with (steady state) *w* plotted on the vertical axis and (steady state) *n* plotted on the horizontal axis to address the following:

i. Is the job creation condition upward-sloping, downward-sloping, perfectly horizontal, perfectly vertical, or is it impossible to tell? All that is needed is a qualitative sketch (numerical values are neither needed nor provided), and clearly label your diagram, providing any necessary explanation.

j. Starting from your sketch in part i, if the (steady state) probability of worker turnover rises (that is, if $p^{TURNOVER}$ increases), what happens to the job creation condition you plotted (i.e., does it rotate, shift, etc?)? All that is needed is a qualitative sketch (numerical values are neither needed nor provided), and clearly label your diagram, providing any necessary explanation.

k. Starting from your sketch in part i, if the (steady state) probability of finding a suitable worker falls (that is, if p^{FIND} decreases), what happens to the job creation condition you plotted (i.e., does it rotate, shift, etc?)? All that is needed is a qualitative sketch (numerical values are neither needed nor provided), and clearly label your diagram, providing any necessary explanation.