Department of Applied Economics Johns Hopkins University

Economics 602
Macroeconomic Theory and Policy
Problem Set 7 Suggested Solutions
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1. **Deriving a Money Demand Function.** Denote by $\phi(c_t, i_t)$ the **real** money demand function. Here you will generate particular functional forms for $\phi(\cdot)$ using the MIU model we have studied.

   In an MIU model, recall that the consumption-money optimality condition can be expressed as
$$\frac{u_{m_t}}{u_{c_t}} = \frac{i_t}{1+i_t},$$
   where u_{m_t} denotes marginal utility with respect to **real** money balances (what was named u_2 in our look at the MIU model) and u_{c_t} denotes marginal utility with respect to consumption (what was named u_1 in our look at the MIU model). In each of the following, you are given a utility function and its associated marginal utility functions. For each case, construct the consumption-money optimality condition and use it to generate the function $\phi(\cdot)$. In each case, your money demand function should end up being an increasing function of c_t and a decreasing function of i_t. (**Note:** Be careful to make the distinction between real money holdings and nominal money holdings. The marginal utility function u_{m_t} is marginal utility with respect to **real** money holdings.)

Solution: For each utility function, we have now written the marginal utility functions u_{c_t} and u_{m_t}. Also note that you are, in each question, being asked to solve for $\frac{M_t}{P_t}$ as a function of c_t and i_t, which is the consumer's real money demand.

a. $u\left(c_t, \frac{M_t}{P_t}\right) = \ln c_t + \ln\left(\frac{M_t}{P_t}\right)$, with $u_{c_t} = \frac{1}{c_t}$ and $u_{m_t} = \frac{1}{M_t / P_t}$.

Solution: Constructing the consumption-money optimality condition with the given functions, we have
$$\frac{u_{m_t}}{u_{c_t}} = \frac{1/(M_t / P_t)}{1/c_t} = \frac{P_t c_t}{M_t} = \frac{i_i}{1+i_t}.$$
Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{c_t(1+i_t)}{i_t}.$$

Thus, the function $\phi(\cdot)$ function is $\phi(c_t, i_t) = \frac{c_t(1+i_t)}{i_t}$, which is increasing in consumption and decreasing in the nominal interest rate, as expected.

b. $u\left(c_t, \frac{M_t}{P_t}\right) = 2\sqrt{c_t} + 2\sqrt{\frac{M_t}{P_t}}$, with $u_{c_t} = \frac{1}{\sqrt{c_t}}$ and $u_{m_t} = \frac{1}{\sqrt{M_t / P_t}}$.

Solution: Proceeding as above, the consumption-money optimality condition is

$$\frac{u_m}{u_{c_t}} = \frac{1/\sqrt{M_t / P_t}}{1/\sqrt{c_t}} = \frac{\sqrt{P_t}\sqrt{c_t}}{\sqrt{M_t}} = \frac{i_i}{1+i_t}.$$

Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{c_t(1+i_t)^2}{i_t^2}$$

(be careful with the algebra here – notice the squared terms in the solution). Thus, the function $\phi(\cdot)$ function is $\phi(c_t, i_t) = \frac{c_t(1+i_t)^2}{i_t^2}$, which is increasing in consumption and decreasing in the nominal interest rate, again as expected.

c. $u\left(c_t, \frac{M_t}{P_t}\right) = c_t^{\sigma} \cdot \left(\frac{M_t}{P_t}\right)^{1-\sigma}$, with $u_{c_t} = \sigma c_t^{\sigma-1}\left(\frac{M_t}{P_t}\right)^{1-\sigma}$ and $u_{m_t} = (1-\sigma)c_t^{\sigma}\left(\frac{M_t}{P_t}\right)^{-\sigma}$.

Solution: The consumption-money optimality condition is

$$\frac{u_m}{u_{c_t}} = \frac{1-\sigma}{\sigma} \cdot \frac{c_t^{\sigma}(M_t / P_t)^{-\sigma}}{c_t^{\sigma-1}(M_t / P_t)^{1-\sigma}} = \frac{i_t}{1+i_t}.$$

After combining exponents, we can write this as

$$\frac{1-\sigma}{\sigma} \cdot c_t \cdot \frac{P_t}{M_t} = \frac{i_i}{1+i_t}.$$

Solving for M_t / P_t, we have

$$\frac{M_t}{P_t} = \frac{1-\sigma}{\sigma} \cdot \frac{c_t(1+i_t)}{i_t}.$$

Thus, the function $\phi(\cdot)$ is $\phi(c_t, i_t) = \frac{1-\sigma}{\sigma} \cdot \frac{c_t(1+i_t)}{i_t}$.

2. **The Keynesian-RBC-New Keynesian Evolution.** Here you will briefly analyze aspects of the evolution in macroeconomic theory over the past 25 years.

a. Describe **briefly** what the Lucas critique is and how/why it led to the demise of (old) Keynesian models.

Solution: The old Keynesian models were large estimated systems of equations, and the estimated coefficients could not (because they were just based on historical observations) take into account how behavior might change if policy changed. In the 1970's, this led to the downfall of such models as policy-makers tried more and more to exploit these relationships, but the "coefficients" began to vary a lot (for some reason…) with policy, eventually causing the profession (through the Lucas critique) to understand that such models really were not all that useful for policy advice after all.

b. Briefly define and describe the neutrality vs. nonneutrality debate surrounding monetary policy today. Which type of shock does this debate concern?

Solution: The RBC view holds that money shocks do not affect real variables (i.e., consumption or GDP) in the economy (neutrality), while the New Keynesian view holds that they do (nonneutrality) because prices take time to adjust (are "sticky").

3. **M1 Money and M2 Money.** Consider an extended version of the infinite-period MIU framework. In addition to stocks and nominal bonds, suppose there are **two** forms of money: M1 and M2. M1 money (which we will denote by M_t^1) and M2 money (which we will denote by M_t^2) both directly affect the representative consumer's utility. The period-*t* utility function is assumed to be

$$u\left(c_t, \frac{M_t^1}{P_r}, \frac{M_t^2}{P_t}\right) = \ln c_t + \ln\frac{M_t^1}{P_t} + \kappa \ln\frac{M_t^2}{P_t},$$

which, note has three arguments. The Greek letter "kappa" (κ) in the utility function is a number between zero and one, $0 \le \kappa \le 1$, over which the representative consumer has no control. The period-t budget constraint of the consumer is

$$P_t c_t + M_t^1 + M_t^2 + B_t + S_t a_t = Y_t + M_{t-1}^1 + (1+i_{t-1}^M)M_{t-1}^2 + (1+i_{t-1})B_{t-1} + (S_t + D_t)a_{t-1},$$

where i_t denotes the nominal interest rate on bonds held between period t and $t+1$ (and hence i_{t-1} on bonds held between $t-1$ and t) and i_t^M denotes the nominal interest rate on M2 money held between period t and $t+1$ (and hence i_{t-1}^M on M2 money held between $t-1$ and t). **Thus, note that M2 money potentially pays interest, in contrast to M1 money, which pays zero interest.**

As always, assume the representative consumer maximizes lifetime utility by optimally choosing consumption and assets (i.e., in this case choosing all four assets optimally).

a. In terms of the notation in this problem, **what is the marginal rate of substitution between real M1 money and real M2 money?** (Hint: You do **not** need to solve a Lagrangian to answer this – all that is required is using the utility function.) Explain the important steps in your argument.

Solution: As always, the MRS is simply the ratio of marginal utilities. The marginal utility function with respect to M1 is $\frac{1/P_t}{M_t^1/P_t}$, and the marginal utility function with respect to M2 is $\frac{\kappa/P_t}{M_t^2/P_t}$ (note that you had to use the chain rule to properly compute these marginal utilities). Constructing the ratio of these and canceling terms, we have the MRS is $\frac{M_t^2}{\kappa M_t^1}$ (i.e., this is the slope at any point of any indifference curve over M1 money and M2 money).

b. A sudden, unexplained change in the value of κ would be interpretable as which of the following: a preference shock, a technology shock, or a monetary policy shock? Briefly explain.

Solution: The parameter κ, as we saw in part a above, affects the MRS between two of the arguments to the utility function (M1 money and M2 money). A change in κ thus affects the slope of the indifference curve, and thus is interpretable as a preference shock. A common error here was to identify a change in κ as a monetary policy shock; this interpretation is incorrect because a monetary policy shock is one that affects money **supply;** here, the shock affects money **demand** (although you didn't have to phrase your arguments exactly this way).

c. Let $\phi^2(c_t, i_t, i_t^M)$ denote the real money demand function for M2 money. Note the three arguments to the function $\phi^2(.)$. Using the first-order conditions of the representative consumer's Lagrangian, generate the function $\phi^2(c_t, i_t, i_t^M)$ (i.e., solve for real M2 money demand as a function of c_t, i_t, and i_t^M). Briefly explain (economically) why i_t^M appears in this money demand function. (**Note:** you must determine yourself which are the relevant first-order conditions needed to create this money demand function – draw on our approach from Chapter 14.)

Solution: Construct the lifetime Lagrangian as usual, and compute the first-order-conditions with respect to c_t, B_t, and M_t^2 (the FOCs on M_t^1 and a_t turn out to be irrelevant in this problem):

$$\frac{1}{c_t} - \lambda_t P_t = 0$$
$$-\lambda_t + \beta\lambda_{t+1}(1+i_t) = 0$$
$$\frac{\kappa / P_t}{M_t^2 / P_t} - \lambda_t + \beta\lambda_{t+1}(1+i_t^M) = 0$$

Substitute the FOC on consumption and bonds into the FOC on M2 money to get, after several algebraic rearrangements,

$$\frac{M_t^2}{P_t} = \frac{\kappa c_t (1+i_t)}{i_t - i_t^M}.$$

Note that this is very similar to the "usual" money demand function obtained when utility is logarithmic (in our "usual" model we implicitly had $\kappa = 0$ and $i_t^M = 0$).

The interest rate i_t^M appears in this money demand function simply because there is an interest benefit of holding this asset, as opposed to no interest in M1 money. As the above money demand function shows, the larger is i_t^M, the larger is M2 money demand. Think of M2 as a savings deposit against which you can write checks, and M1 money as cash. Cash earns you zero interest, whereas a savings deposit earns you some positive interest; on the other hand, cash is accepted everywhere, but checks against your savings deposit are not accepted everywhere (i.e., savings deposits are less liquid than cash).