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Economics 602
Macroeconomic Theory and Policy
Problem Set 9 Suggested Solutions
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1. Stock, Bonds, "Bills," and the Financial Accelerator. In this problem, you will study an enriched version of the accelerator framework we studied in class. As in our basic analysis, we continue to use the two-period theory of firm profit maximization as our vehicle for studying the effects of financial-market developments on macroeconomic activity. However, rather than supposing it is just "stock" that is the financial asset at firms' disposal for facilitating physical capital purchases, we will now suppose that **both "stock" and "bonds" are at firms' disposal for facilitating physical capital purchases.**

Before describing more precisely the analysis you are to conduct, a deeper understanding of "bond markets" is required. In "normal economic conditions," (i.e, in or near a "steady state," in the sense we first discussed in Chapter 8), it is usually sufficient to think of all bonds of various maturity lengths in a highly simplified way: by supposing that they are all simply one-period face-value = 1 bonds with the same nominal interest rate. Recall, in fact, that this is how our basic discussion of monetary policy proceeded. In "unusual" (i.e., far away from steady state) financial market conditions, however, it can become important to distinguish between different types of bonds and hence different types of nominal interest rates on those bonds.

You may have seen discussion in the press about central banks, such as the U.S. Federal Reserve, considering whether or not to "begin buying bonds" as a way of conducting policy. Viewed through the standard lens of how to understand open-market operations, this discussion makes no sense because in the standard view, central banks **already do** buy (and sell) "bonds" as the mechanism by which they conduct open-market operations!

A difference that becomes important to understand during unusual financial market conditions is that open-market operations are conducted using the **shortest-maturity** "bonds" that the Treasury sells, of duration one month or shorter. In the lingo of finance, this type of "bond" is called a "Treasury bill." The term "Treasury bond" is usually used to refer to **longer-maturity** Treasury securities – those that have maturities of one, two, five, or more years. These longer-maturity Treasury "bonds" have typically **not** been assets that the Federal Reserve buys and sells as regular practice; buying such longer-maturity bonds is/has not been the usual way of conducting monetary policy.

In the ensuing analysis, part of the goal will be to understand/explain why policy-makers are currently considering this option. Before beginning this analysis, though, there is more to understand.

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In private-market borrower/lender relationships, longer-maturity Treasury bonds ("bonds") are typically allowed to be used just like stocks in financing firms' physical capital purchases.[1] We can capture this idea by enriching the financing constraint in our financial accelerator framework to read:

$$P_1 \cdot (k_2 - k_1) = R^S \cdot S_1 \cdot a_1 + R^B \cdot P_1^b \cdot B_1 .$$

The left hand side of this richer financing constraint is the same as the left hand side of the financing constraint we considered in our basic theory (and the notation is identical, as well – refer to your notes for the notational definitions).

The right hand side of the financing constraint is richer than in our basic theory, however. The market value of "stock," $S_1 a_1$, still affects how much physical investment firms can do, scaled by the government regulation R^S. **In addition, now the market value of a firm's "bond-holdings" (which, again, means long-maturity government bonds) also affects how much physical investment firms can do,** scaled by the government regulation R^B. The notation here is that B_1 is a firm's holdings of nominal bonds ("long-maturity") at the end of period 1, and P_1^b is the nominal price of that bond during period 1. Note that R^B and R^S need not be equal to each other.

In the context of the two-period framework, the firm's two-period discounted profit function now reads:

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 + B_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 - P_1^b B_1$$
$$+ \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} + \frac{B_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i} - \frac{P_2^b B_2}{1+i}$$

The new notation compared to our study of the basic accelerator mechanism is the following: B_0 is the firm's holdings of nominal bonds (**which have face value = 1)** at the start of period one, B_1 is the firm's holdings of nominal bonds (**which have face value = 1**) at the end of period one, and B_2 is the firm's holdings of nominal bonds (**which have face value = 1)** at the end of period two.

Note that period-2 profits are being discounted by the nominal interest rate i: in this problem, we will consider i to be the "Treasury bill" interest rate (as opposed to the "Treasury bond" interest rate). The Treasury-**bill** interest rate is the one the Federal Reserve usually (i.e., in "normal times") controls. We can **define** the nominal interest rate on Treasury **bonds** as

$$i^{BOND} = \frac{1}{P_1^b} - 1 \left(\Leftrightarrow P_1^b = \frac{1}{1 + i^{BOND}} \right)$$

Thus, note that i^{BOND} and i need not equal each other.

[1] Whereas, for various institutional and regulatory reasons, very short-term Treasury assets ("T-bills") are typically not allowed to be used in financing firms' physical capital purchases.

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The rest of the notation above is just as in our study of the basic financial accelerator framework. Finally, because the economy ends at the end of period 2, we can conclude (as usual) that $k_3 = 0$, $a_2 = 0$, and $B_2 = 0$.

With this background in place, you are to analyze a number of issues.

a. Using λ as your notation for the Lagrange multiplier on the financing constraint, construct the Lagrangian for the representative firm's (two-period) profit-maximization problem.

Solution: The Lagrangian, which by now should be extremely straightforward to construct, is

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 + B_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 - P_1^b B_1$$
$$+\frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} + \frac{B_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i} - \frac{P_2^b B_2}{1+i}$$
$$+\lambda\left[R^S S_1 a_1 + R^B P_1^b B_1 - P_1(k_2 - k_1)\right]$$

b. Based on this Lagrangian, compute the first-order condition with respect to nominal bond holdings at the end of period 1 (i.e., compute the FOC with respect to B_1). **(Note:** This FOC is critical for much of the analysis that follows, so you should make sure that your work here is absolutely correct.)

Solution: Based on Lagrangian above, the FOC with respect to B_1 is

$$-P_1^b + \frac{1}{1+i} + \lambda R^B P_1^b = 0.$$

c. Recall that in this enriched version of the accelerator framework, the nominal interest rate on "Treasury bills," i, and the nominal interest rate on "Treasury bonds," i^{BOND}, are potentially different from each other. If financing constraints do NOT at all affect firms' investment in physical capital, how does i^{BOND} compare to i? Specifically, is i^{BOND} equal to i, is i^{BOND} smaller than i, is i^{BOND} larger than i, or is it impossible to determine? Be as thorough in your analysis and conclusions as possible (i.e., tell us as much about this issue as you can!). Your analysis here should be based on the FOC on B_1 computed in part b above. (**Hint:** if financing constraints "don't matter," what is the value of the Lagrange multiplier λ?)

Solution: As discussed in detail in class, financing constraints are said to "not matter" (in the context of the accelerator framework) when the value of the Lagrange multiplier is zero, $\lambda = 0$. Inserting this value for the multiplier in the FOC derived in part b, we have that

$$P_1^b = \frac{1}{1+i}.$$

Keeping in mind that in this problem we are distinguishing between i and i^{BOND}, this last expression can be written as

$$\frac{1}{1+i^{BOND}} = \frac{1}{1+i},$$

from which it is obvious that $i = i^{BOND}$. Thus, in "normal economic conditions" (i.e., when $\lambda = 0$), the nominal interest rates on "Treasury bills" and "Treasury bonds" are exactly equal. This analytical result in fact justifies the usual practice of treating all bonds as "the same" – in normal economic conditions, their interest rates are (roughly) equalized. (Indeed, if we introduced even longer maturity bonds into our framework – two-period bonds, three-period bonds, five-period bonds, etc. – we would be led to same conclusion, that all of their interest rates are equal to each other, provided that financing constraints don't affect macroeconomic outcomes – although "impatience" introduces another caveat into this, but we have ignored impatience issues in this problem.)

d. If financing constraints DO affect firms' investment in physical capital, how does i^{BOND} compare to i? Specifically, is i^{BOND} equal to i, is i^{BOND} smaller than i, is i^{BOND} larger than i, or is it impossible to determine? Furthermore, if possible, use your solution here as a basis for justifying whether or not it is appropriate in "normal economic conditions" to consider both "Treasury bills" and "Treasury bonds" as the "same" asset. Be as thorough in your analysis and conclusions as possible. Once again, your analysis here should be based on the FOC on B_1 computed in part b above. **(Note:** the government regulatory variables R^S and R^B are both strictly positive – that is, neither can be zero or less than zero).

Solution: From the FOC on B_1 computed in part b, and now **without** imposing $\lambda = 0$, we can perform the following algebraic rearrangements:

$$-P_1^b + \frac{1}{1+i} + \lambda R^B P_1^b = 0$$

$$\lambda = \left[P_1^b - \frac{1}{1+i}\right] \cdot \frac{1}{R^B P_1^b}$$

$$\lambda = \left[1 - \frac{1}{P_1^b}\frac{1}{1+i}\right] \cdot \frac{1}{R^B}$$

$$\lambda = \left[1 - \frac{1+i^{BOND}}{1+i}\right] \cdot \frac{1}{R^B}$$

$$\lambda = \left[\frac{1+i-(1+i^{BOND})}{1+i}\right] \cdot \frac{1}{R^B}$$

$$\lambda = \left[\frac{i-i^{BOND}}{1+i}\right] \cdot \frac{1}{R^B}$$

This final expression shows that, if financing constraints "matter" (which means that $\lambda \neq 0$), then clearly $i \neq i^{BOND}$. Without knowing more about "how" financial market conditions are affecting investment behavior – that is, whether financing conditions are "tight" or "loose" (which would govern the sign of the multiplier λ), it is impossible to say anything more about how the T-bill interest rate and the T-bond interest rate compared to each other. But, regardless, it is clear that it is **not** appropriate to consider the two interest rates as being **identical** in this case.

The above analysis was framed in terms of nominal interest rates; the remainder of the analysis is framed in terms of real interest rates.

e. By computing the first-order condition on firms' stock-holdings at the end of period 1, a_1, and following exactly the same algebra as presented in class, we can express the Lagrange multiplier λ as

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R^S}. \tag{1.1}$$

Use the first-order condition on B_1 you computed in part b above to derive an analogous expression for λ except in terms of the real interest rate on bonds (i.e., r^{BOND}) and R^B (rather than R^S). (**Hint:** Use the FOC on B_1 you computed in part b above and follow a very similar set of algebraic manipulations as we followed in class.)

Solution: Based on the derivations in part d above, this step simply requires applying the Fisher relation a couple of times. Specifically, let's start again with the final condition obtained in part d above

$$\lambda = \left[\frac{i - i^{BOND}}{1+i}\right] \cdot \frac{1}{R^B}$$

and rewrite it as

$$\lambda = \left[\frac{1+i-(1+i^{BOND})}{1+i}\right] \cdot \frac{1}{R^B}.$$

Next, multiply and divide the term inside square brackets by (1+π) (which of course simply means we're multiplying by one), which gives

$$\lambda = \left[\frac{\frac{1+i}{1+\pi} - \frac{1+i^{BOND}}{1+\pi}}{\frac{1+i}{1+\pi}}\right] \cdot \frac{1}{R^B}$$

By the Fisher relation, we can express this as

$$\lambda = \left[\frac{1+r-(1+r^{BOND})}{1+r}\right] \cdot \frac{1}{R^B},$$

or, finally,

$$\lambda = \left[\frac{r - r^{BOND}}{1+r}\right] \cdot \frac{1}{R^B},$$

which is obviously similar to the type of condition we derived in class regarding stock financing.

f. Compare the expression you just derived in part e with expression (1.1). Suppose $r = r^{STOCK}$. If this is the case, is r^{BOND} equal to r, is r^{BOND} smaller than r, is r^{BOND} larger than r, or is it impossible to determine? Furthermore, in this case, does the financing constraint affect firms' physical investment decisions? Briefly justify your conclusions and provide brief explanation.

Solution: The expression derived in part e and expression (1.1) both feature λ on the left hand side. We can thus obviously set them equal to each other, giving us

$$\left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R^S} = \left[\frac{r - r^{BOND}}{1+r}\right] \cdot \frac{1}{R^B}.$$

Although you did not have to perform the next algebraic step (i.e., you could conduct the ensuing logical analysis based just on this last expression), we can multiply this entire expression through by $1+r$ and then also multiply the entire expression by R^S, which would give us

$$r - r^{STOCK} = (r - r^{BOND}) \cdot \frac{R^S}{R^B},$$

which makes obvious what the consequence of $r = r^{STOCK}$ is. If $r = r^{STOCK}$, obviously it must also be that $r = r^{BOND}$ (because you are told that R^S can**not** be zero).

Thus, if the returns on stocks ("risky assets") are equal to the return on physical assets (r), the return on bonds ("safe financial assets") are also equal to the return on physical assets (r). This is essentially just a statement of the Fisher relation – recall from Chapter 14 that one way to understand/interpret the Fisher equation is that it says the returns on "safe" and "risky" assets are equal to each other (the "no-arbitrage" relationship). Here, the underlying view is that the returns on **all types of bonds** are "riskless" ("safe") , just as is the returns on physical capital.

g. Through late 2008, suppose that $r = r^{STOCK}$ was a reasonable description of the U.S. economy for the preceding 20+ years. In late 2008, r^{STOCK} fell dramatically below r, which, as we studied in class, would cause the financial accelerator effect to begin. Suppose government policy-makers, both fiscal policy-makers and monetary policy-makers, decide that they need to intervene in order to try to choke off the accelerator effect. Furthermore, suppose that there is no way to change either R^S or R^B (because of coordination delays amongst various government agencies, perhaps). Using all of your preceding analysis as well as drawing on what we studied in class, explain why "buying bonds" (which, again, means long-maturity bonds in the sense described above) might be a sound strategy to pursue. (**Note:** The analysis here is largely **not** mathematical. Rather, what is required is an careful logical progression of thought that explains why buying bonds might be a good idea.)

Solution: As we discussed in class, one way of offsetting the feedback effects of a decline in financial market returns is to relax financial market regulations – increasing R^S and/or R^B in this case. The reason this may be helpful is that, all else equal, it would serve to lower λ (examine the conditions derived above), which is, analytically, where the problems can be traced to (ie, the fact that financing constraints "matter"). From the financing constraint itself, it is obvious that raising R^S

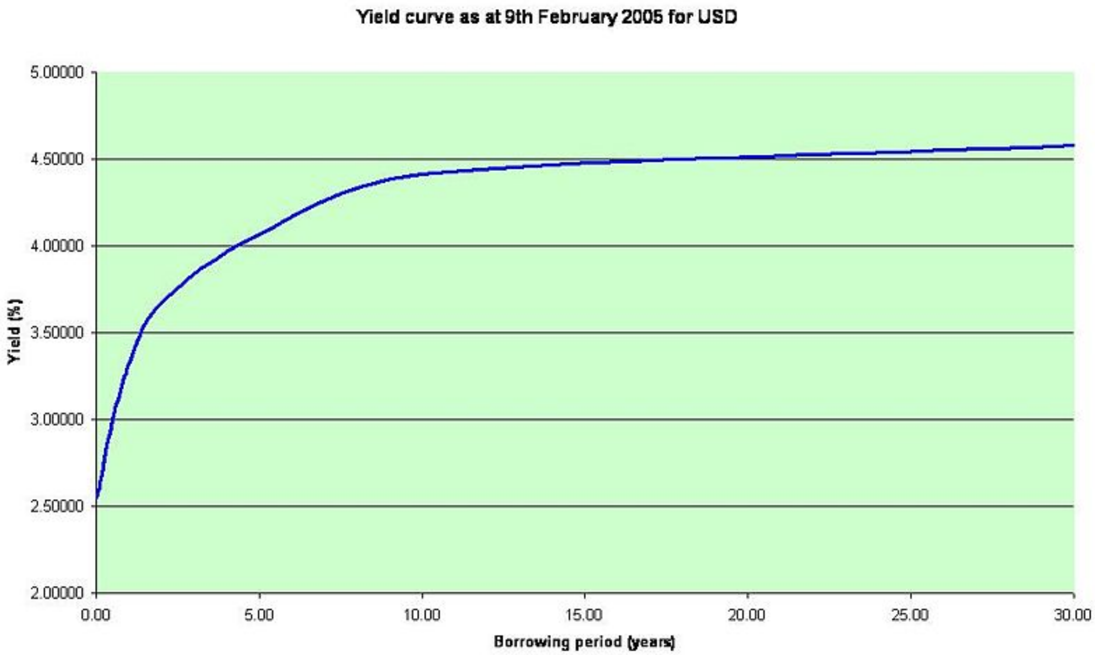
and/or R^B increases the "effective" market value of firms' collateralizeable financial assets (recall the basic information asymmetry problems that underlie this financing constraint):

$$P_1 \cdot (k_2 - k_1) = R^S \cdot S_1 \cdot a_1 + R^B \cdot P_1^b \cdot B_1 .$$

That is, raising R^S and/or R^B increases the right hand side of the financing constraint. But if that is infeasible for institutional or political other reasons, another policy intervention that has the same effect is to try to raise **any** of the other components of the right hand side of the financing constraint: including government efforts to try to raise the price of bonds by directly buying them in markets (i.e., the increased demand for bonds in bond markets should, all else equal, raise the price of bonds).

This is beyond the scope of this question, but this type of analysis can shed light on a host of policy proposals and programs that are being/have been discussed the past year: many of them share the broad goal of trying to raise the "effective: market value of the private sector's collateralizeable financial assets. This could be achieved by some combination of direct government purchases of a variety of financial assets (stocks, bonds), simply "giving" firms "more assets" (i.e., directly giving them more a and/or more B), allowing "new types" of financial assets to be used for collateral purposes (i.e., adding a third asset to the right hand side of the financing constraint, a fourth asset to the right hand side of the financing constraint, etc.): broadly speaking, it's all about raising the right hand side of the financing constraint above!

2. **The Yield Curve.** An important indicator of markets' beliefs/expectations about the future path of the macroeconomy is the "yield curve," which, simply put, describes the relationship between the maturity length of a particular bond (recall that bonds come in various maturity lengths) and the per-year interest rate on that bond. A bond's "yield" is alternative terminology for its interest rate. A sample yield curve is shown in the following diagram:



This diagram plots the yield curve for U.S. Treasury bonds that existed in markets on February 9, 2005: as it shows, a 5-year Treasury bond on that date carried an interest rate of about 4 percent, a 10-year Treasury bond on that date carried an interest rate of about 4.4 percent, and a 30-year Treasury bond on that date carried an interest rate of about 4.52 percent.

Recall from our study of bond markets that prices of bonds and nominal interest rates on bonds are negatively related to each other. The yield curve is typically discussed in terms of nominal interest rates (as in the graph above). However, because of the inverse relationship between interest rates on bonds and prices of bonds, the yield curve could equivalently be discussed in terms of the prices of bonds.

In this problem, you will use an enriched version of our infinite-period monetary framework from Chapter 14 to study how the yield curve is determined. Specifically, rather than assuming the representative consumer has only one type of bond (a one-period bond) he can purchase, we will assume the representative consumer has several types of bonds he can purchase – a one-period bond, a two-period bond, and, in the later parts of the problem, a three-period bond.

Let's start just with two-period bonds. We will model the two-period bond in the simplest possible way: in period t, the consumer purchases B_t^{TWO} units of two-period bonds, each of which has a market price $P_t^{b,TWO}$ and a face value of one (i.e., when the two-period bond pays off, it pays back one dollar). **The defining feature of a two-period bond is that it pays back its face value *two* periods after purchase** (indeed, hence the term "two-period bond"…). The one-period bond is just as we have discussed in class and in Chapter 14.

Mathematically, then, suppose (just as in Chapter 14) that the representative consumer has a lifetime utility function starting from period t

$$\ln c_t + \ln\left(\frac{M_t}{P_t}\right) + \beta \ln c_{t+1} + \beta \ln\left(\frac{M_{t+1}}{P_{t+1}}\right) + \beta^2 \ln c_{t+2} + \beta^2 \ln\left(\frac{M_{t+2}}{P_{t+2}}\right) + \beta^3 \ln c_{t+3} + \beta^3 \ln\left(\frac{M_{t+3}}{P_{t+3}}\right)...,$$

and his period-t budget constraint is given by

$$P_t c_t + P_t^b B_t + P_t^{b,TWO} B_t^{TWO} + M_t + S_t a_t = Y_t + M_{t-1} + B_{t-1} + B_{t-2}^{TWO} + (S_t + D_t) a_{t-1}.$$

(Based on this, you should know what the period t+1 and period t+2 and period t+3, etc. budget constraints look like). This budget constraint is identical to that in Chapter 14, except of course for the terms regarding two-period bonds. **Note carefully the timing on the right hand side – in accordance with the defining feature of a two-period bond, in period *t*, it is B_{t-2}^{TWO} that pays back its face value.** The rest of the notation is just as in Chapter 14, including the fact that the subjective discount factor (i.e., the measure of impatience) is $\beta < 1$.

a. Qualitatively represent the yield curve shown in the diagram above in terms of **prices of bonds** rather than interest rates on bonds. That is, with the same maturity lengths on the horizontal axis, plot (qualitatively) on the vertical axis the prices associated with these bonds.

Solution: With maturity lengths plotted on the horizontal axis, the yield curve **in terms of bond prices** is downward-sloping. This follows simply because of the inverse relationship between bond prices and interest rates. The yield curve shown above is in terms of interest rates and is strictly increasing; hence the associated yield curve in terms of prices must be strictly decreasing.

b. Based on the utility function and budget constraint given above, set up an appropriate Lagrangian in order to derive the representative consumer's first-order conditions with respect to **both** B_t **and** B_t^{TWO} (as usual, the analysis is being conducted from the perspective of the very beginning of period t). Define any auxiliary notation that you need in order to conduct your analysis.

Solution: The only two first-order conditions that you needed here are those on B_t and B_t^{TWO}. Denoting by λ_t the Lagrange multiplier on the period-t budget constraint and by λ_{t+1} the Lagrange multiplier on the period-t+1 budget constraint, the two first-order conditions, respectively, are

$$-\lambda_t P_t^b + \beta \lambda_{t+1} = 0$$

and

$$-\lambda_t P_t^{b,TWO} + \beta^2 \lambda_{t+2} = 0.$$

Note well the t+2 time subscripts in the second expression; this follows from the fact the a two-period bond purchased in period t does not repay its promised face value until period t+2. (Refer back to Problem Set 4 for an analogous stock-pricing model in which stocks took two periods to pay off their capital gains and dividends.)

c. Using the two first-order conditions you obtained in part b, construct a relationship between the price of a two-period bond and the price of a one-period bond. Your final relationship should be of the form $P_t^{b,TWO} = \ldots$, and **on the right-hand-side of this expression should appear** (potentially among other things), P_t^b. (Hint: in order to get P_t^b into this expression, you may have to multiply and/or divide your first-order conditions by appropriately-chosen variables.)

Solution: From the first expression above, we have, as usual that $P_t^b = \frac{\beta\lambda_{t+1}}{\lambda_t}$. From the second expression above, we analogously can obtain $P_t^{b,TWO} = \frac{\beta^2\lambda_{t+2}}{\lambda_t}$. We can rewrite this expression for the price of a two-period bond as

$$P_t^{b,TWO} = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}}\frac{\beta\lambda_{t+1}}{\lambda_t},$$

in which we have simply multiplied and divided the preceding expression by λ_{t+1} (i.e., we have multiplied by one, always a valid mathematical operation). The final term on the far right-hand-side is nothing more than the price of a one-period bond, so we can write

$$P_t^{b,TWO} = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}}P_t^b,$$

which satisfies the form of the relationship you were asked to derive. We can actually boil this down further, though. Note that **the price of one-period bond purchased *in* period t+2 would be given by $P_{t+1}^b = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}}$,** which follows from optimization with respect to period t+1 **one-period** bond holdings. Using this expression in the period-t price of a **two-period** bond, we thus obtain

$$P_t^{b,TWO} = P_{t+1}^b P_t^b,$$

which is a key idea in finance theory: the price of a multi-period asset (bond) is nothing more than the **product of the prices of two consecutive one-period assets** (bond).

d. Suppose that the optimal **nominal expenditure on consumption** (Pc) is equal to 1 in every period. Using this fact, is the price of a two-period bond greater than, smaller than, or equal to the price of a one-year bond? If it is impossible to tell, explain why; if you can tell, be as precise as you can be about the relationship between the prices of the two bonds. (Hint: you may need to invoke the consumer's first-order condition on consumption)

Solution: Start with the relationship $P_t^{b,TWO} = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}} P_t^b$ derived above. If nominal consumption expenditures are constant (and equal to one) every period, this means that $\lambda = 1$ every period. (This conclusion follows from the fact that the FOC with respect to consumption is $1/c_t - \lambda_t P_t = 0$ in every period, which can be rearranged to $\lambda_t = \frac{1}{P_t c_t}$). If the multiplier is one every period, we immediately have

$$P_t^{b,TWO} = \beta P_t^b.$$

Because $\beta < 1$, we conclude $P_t^{b,TWO} < P_t^b$.

e. Now suppose there is also a three-period bond. A three-period bond purchased in any given period does not repay its face value (also assumed to be 1) until *three* periods after it is purchased. The period-t budget constraint, now including one-, two-, and three-period bonds, is given by

$$P_t c_t + P_t^b B_t + P_t^{b,TWO} B_t^{TWO} + P_t^{b,THREE} B_t^{THREE} + M_t + S_t a_t = Y_t + M_{t-1} + B_{t-1} + B_{t-2}^{TWO} + B_{t-3}^{THREE} + (S_t + D_t) a_{t-1},$$

where B_t^{THREE} is the quantity of three-period bonds purchased in period t and $P_t^{b,THREE}$ its associated price. Following the same logical steps as in parts b, c, and d above (and continuing to assume that nominal expenditure on consumption (Pc) is equal to one in period every period), is the price of a three-year bond greater than, smaller than, or equal to the price of a two-year bond? If it is impossible to tell, explain why; if you can tell, be as precise as you can be about the relationship between the prices of the two bonds. (Hint: you may need to invoke the consumer's first-order condition on consumption).

Solution: Extending the Lagrangian from above, the first-order condition with respect to B_t^{THREE} is

$$-\lambda_t P_t^{b,THREE} + \beta^3 \lambda_{t+3} = 0,$$

which can be rearranged to yield $P_t^{b,THREE} = \frac{\beta^3 \lambda_{t+3}}{\lambda_t}$. Just like in part c above, by cleverly multiplying by one, we can express this as

$$P_t^{b,THREE} = \frac{\beta^2 \lambda_{t+3}}{\lambda_{t+2}} \frac{\beta\lambda_{t+2}}{\lambda_{t+1}} \frac{\beta\lambda_{t+1}}{\lambda_t},$$

which, in exactly the same way as in part c, we can express in terms of chained one-period bond prices,

$$P_t^{b,THREE} = \frac{\beta\lambda_{t+3}}{\lambda_{t+2}} P_{t+1}^b P_t^b .$$

If the Lagrange multiplier λ is constant every period, we can conclude the price of a three-period bond is smaller than the price of a two-period bond (which in turn, from part c, is smaller than the price of a one-period bond). This again follows because $\beta < 1$.

f. Suppose that $\beta = 0.95$. Using your conclusions from parts d and e, plot a yield curve in terms of bond prices (obviously, you can plot only three different maturity lengths here).

Solution: Based on the analyses in parts d and e, the price of bonds is clearly negatively-related to its maturity length, hence the yield curve in terms of prices is strictly decreasing. This is just as your sketch of the empirical yield curve in part a.

g. What is the single most important reason (economically, that is) for the shape of the yield curve you found in part f? (This requires only a brief, qualitative/conceptual response.)

Solution: Re-examining our conclusions/analyses in parts d, e, and f, the sole reason we were able to reach the conclusions we reached in each of those parts was the fact that $\beta < 1$. Thus, the idea of impatience and its effects on the macroeconomy rears its head again, this time with respect to bond prices of different maturities. The conceptual idea is simple: because of impatience, the longer a bond purchaser must wait to receive a given face value, the less he will be willing to pay for it today (and this is reflected in bond market prices through the bond demand function for different maturity bonds).