Department of Applied Economics Johns Hopkins University

Economics 602

Macroeconomic Theory and Policy

Final Exam

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NAME: ______________________________

The Exam has a total of four (4) problems and pages numbered one (1) through eighteen (18). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use two pages (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 25**
Problem 3	**/ 30**
Problem 4	**/ 20**
TOTAL	**/ 100**

Problem 1: "Hyperbolic Impatience" and Stock Prices (25 points). In this problem you will study a slight extension of the infinite-period economy from Chapter 8. Specifically, suppose the representative consumer has a lifetime utility function given by

$$u(c_t) + \gamma\beta u(c_{t+1}) + \gamma\beta^2 u(c_{t+2}) + \gamma\beta^3 u(c_{t+3}) + ...,$$

in which, as usual, $u(.)$ is the consumer's utility function in any period and β is a number between zero and one that measures the "normal" degree of consumer impatience. **The number γ (the Greek letter "gamma," which is the new feature of the analysis here) is also a number between zero and one, and it measures an "additional" degree of consumer impatience, but one that ONLY applies between period t and period t+1.[1] This latter aspect is reflected in the fact that the factor γ is NOT successively raised to higher and higher powers as the summation grows.**

The rest of the framework is exactly as studied in Chapter 8: a_{t-1} is the representative consumer's holdings of stock at the beginning of period *t,* the nominal price of each unit of stock during period *t* is S_t , and the nominal dividend payment (per unit of stock) during period *t* is D_t . Finally, the representative consumer's consumption during period *t* is c_t and the nominal price of consumption during period *t* is P_t. As usual, analogous notation describes all these variables in periods *t*+1, *t*+2, etc.

The Lagrangian for the representative consumer's utility-maximization problem (starting from the perspective of the beginning of period *t*) is

$$\begin{aligned}
&u(c_t) + \gamma\beta u(c_{t+1}) + \gamma\beta^2 u(c_{t+2}) + \gamma\beta^3 u(c_{t+3}) + ...\\
&+\lambda_t \left[Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t \right]\\
&+\gamma\beta\lambda_{t+1} \left[Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - S_{t+1} a_{t+1} \right]\\
&+\gamma\beta^2\lambda_{t+2} \left[Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2} c_{t+2} - S_{t+2} a_{t+2} \right]\\
&+\gamma\beta^3\lambda_{t+3} \left[Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3} c_{t+3} - S_{t+3} a_{t+3} \right]\\
&+...
\end{aligned}$$

NOTE CAREFULLY WHERE THE "ADDITIONAL" IMPATIENCE FACTOR γ APPEARS IN THE LAGRANGIAN.

(OVER)

[1] The idea here, which goes under the name "hyperbolic impatience," is that in the "very short run" (i.e., between period t and period t+1), individuals' degree of impatience may be different from their degree of impatience in the "slightly longer short run" (i.e., between period t+1 and period t+2, say). "Hyperbolic impatience" is a phenomenon that routinely recurs in laboratory experiments in experimental economics and psychology, and has many far-reaching economic, financial, policy, and societal implications.

Problem 1 continued

a. **(3 points)** Compute the first-order conditions of the Lagrangian above with respect to **both** a_t **and** a_{t+1}. (**Note:** There is no need to compute first-order conditions with respect to any other variables.)

b. **(4 points)** Using the first-order conditions you computed in part a, construct two distinct stock-pricing equations, one for the price of stock in period *t*, and one for the price of stock in period *t*+1. Your final expressions should be of the form $S_t = ...$ and $S_{t+1} = ...$ (**Note:** It's fine if your expressions here contain Lagrange multipliers in them.)

Problem 1 continued

For the remainder of this problem, suppose it is known that $D_{t+1} = D_{t+2}$, and that $S_{t+1}=S_{t+2}$, and that $\lambda_t = \lambda_{t+1} = \lambda_{t+2}$.

c. **(5 points).** Does the above information necessarily imply that the economy is in a steady-state? **Briefly and carefully explain why or why not; your response should make clear what the definition of a "steady state" is. (Note:** To address this question, it's possible, though not necessary, that you may need to compute other first-order conditions besides the ones you have already computed above.)

d. **(5 points)** Based on the above information and your stock-price expressions from part b, can you conclude that the period-t stock price (S_t) is higher than S_{t+1}, lower than S_{t+1}, equal to S_{t+1}, or is it impossible to determine? **Briefly and carefully explain the economics (i.e., the economic reasoning, not simply the mathematics) of your finding.**

Problem 1 continued

Now also suppose that the utility function in every period is $u(c) = \ln c$, and also that the **real interest rate is zero in every period.**

e. **(4 points) Based on the utility function given, the fact that $r = 0$, and the basic setup of the problem described above, construct two marginal rates of substitution (MRS):** the MRS between period-t consumption and period-t+1 consumption, **and** the MRS between period-t+1 consumption and period-t+2 consumption.

f. **(4 points – Harder)** Based on the two MRS functions you computed in part e and on the fact that $r = 0$ in every period, determine which of the following two consumption growth rates

$$\frac{c_{t+1}}{c_t} \quad \text{OR} \quad \frac{c_{t+2}}{c_{t+1}}$$

is larger. That is, is the consumption growth rate between period t and period t+1 (the fraction on the left) expected to be larger than, smaller than, or equal to the consumption growth rate between period t+1 and period t+2 (the fraction on the right), or is it impossible to determine? **Carefully explain your logic, and briefly explain the economics (i.e., the economic reasoning, not simply the mathematics) of your finding.**

Problem 2: The Cash-in-Advance (CIA) Framework (25 points). A popular alternative to the money in the utility function (MIU) framework is one in which money holdings directly facilitate transactions – that is, provide a medium of exchange role, one of the basic functions of "money" that the MIU framework captures in only a shortcut form.

Suppose that in any given time period, there are two "types" of consumption goods, "cash goods" and "credit goods." Cash goods, denoted by c_{1t}, are goods whose purchase in period t **requires** money, while credit goods, denoted by c_{2t}, are goods whose purchase in period t does not require money (i.e., they can be bought "on credit"). The market nominal price of each type of good is identical, P_t.

The representative consumer consumes both cash and credit goods. Specifically, suppose the period-t utility function is $u(c_{1t}, c_{2t}, n_t)$, with n_t denoting the individual's **labor** during period t. (As in Chapter 2, suppose that total hours available in any given time period is 168, and the only possible uses of time are labor or leisure.)

The consumer's period-t budget constraint is

$$P_t c_{1t} + P_t c_{2t} + M_t + P_t^b B_t = P_t w_t n_t + M_{t-1} + B_{t-1},$$

Income is earned from labor supply (with w_t denoting the market determined **real** wage in period t, which is taken as given by the individual), and, for simplicity, suppose there are no stock markets (hence one-period riskless bonds and money markets are the only two asset markets). The consumer's bond and money holdings at the start of period t are B_{t-1} and M_{t-1}, and at the end of period t are B_t and M_t. The individual's budget constraints for period t+1, t+2, … are identical to the above, with the time subscripts appropriately updated. As always, suppose the representative consumer's subjective discount factor between any pair of consecutive time periods is $\beta \in (0,1)$.

In addition to the budget constraint, in each time period the representative consumer **also has a "cash in advance" constraint,**

$$P_t c_{1t} = M_t.$$

The cash-in-advance (CIA) constraint captures the idea that in order to purchase some goods, a certain amount of money (or more generally, "liquidity" such as checkable deposits) has to be held. In principle, the CIA constraint is an inequality constraint (specifically, $P_t c_{1t} \le M_t$), **but in analyzing this problem, you are to assume that it always holds with equality, as written above.** From the standpoint of the analysis you will conduct, because the CIA is technically an inequality constraint, you **may NOT substitute the CIA constraint into the budget constraint.**

Problem 2 continued

The consumer's budget constraints and CIA constraints for period t+1, t+2, … are identical to the above, with the time subscripts appropriately updated.

a. **(4 points)** Set up an appropriate sequential Lagrangian from the perspective of the beginning of period t. Define any new notation you introduce.

b. **(6 points)** Based on the Lagrangian constructed in part a, derive the first-order conditions (FOCs) with respect to period t's choices, c_{1t}, c_{2t}, n_t, B_t, and M_t. Define any new notation you introduce.

Problem 2 continued

c. **(5 points)** Using the FOCs from above, derive the cash good/credit good optimality condition, which should have a final form $\frac{u_1(c_{1t}, c_{2t}, n_t)}{u_2(c_{1t}, c_{2t}, n_t)} = f(i_t)$. **Note that $f(i_t)$ is a function that depends ONLY on the nominal interest rate. You are to determine $f(i_t)$ as part of this problem;** there should be no other variables or parameters at all on the right-hand-side of the cash good/credit good optimality condition you derive. Show clearly the important steps in the algebra.

Problem 2 continued

d. **(5 points)** Suppose now that the utility function is $u(c_{1t}, c_{2t}, n_t) = \ln c_{1t} + \ln c_{2t} + v(n_t)$, in which $v(n_t)$ is some unspecified function of labor. **Taking into account the fact that the CIA constraint holds with equality at the optimal choice,** derive, based on this utility function and your work above, the **real money demand function,**

$$\frac{M_t}{P_t} = \ldots$$

where the term on the right-hand side is for you to determine. Show clearly the important steps in the algebra. **Also,** qualitatively plot this relationship in a diagram with M/P on the horizontal axis and i on the vertical axis, clearly labeling the two axes.

Problem 2 continued

e. **(5 points)** Recall that **in the MIU framework, the consumption-money optimality condition was**

$$\frac{\text{marginal utility of real money holdings}}{\text{marginal utility of consumption}} = \frac{i_t}{1+i_t}.$$

Compare, in terms of economics, this optimality condition to the optimality condition you obtained in part c above for the CIA framework **by briefly commenting on the similarities and differences between the results predicted by the two frameworks. Note:** this does **not** mean restate in words the mathematics; rather, offer two or three thoughts/critiques/etc. on how and why the two frameworks do or not capture the same ideas, how and why the two frameworks perhaps are or are not essentially identical to each other, and so on.

Problem 3: Elasticity of Labor Supply and Fiscal Policy. (30 points). Consider the static (i.e., one-period) consumption-leisure framework. In quantitative and policy applications that use this framework, a commonly-used utility function is

$$u(c,l) = \ln c - \frac{\theta}{1+1/\psi}(168-l)^{1+1/\psi},$$

in which *c* denotes consumption, *l* denotes the number of hours (in a week) spent in leisure, and ψ and θ (the Greek letters "psi" and "theta," respectively) are **constants** (even though we will not assign any numerical value to them) in the utility function. The representative individual has no control over either ψ or θ, and both $\psi > 0$ and $\theta > 0$.

Labor is measured as $n = 168 - l$, the **real wage** is denoted by *w*, and the labor-income tax rate is denoted by *t*. The **take-home rate** (the fraction of labor income that an individual keeps) can thus be defined as $S = (1\text{-}t)$. Finally, expressed in real terms, the representative individual's budget constraint is

$$c = (1-t)wn.$$

Thus, this is **exactly** the consumption-leisure framework studied in Chapter 2, with now a particular functional form for $u(c,l)$.

The **consumption-leisure optimality condition** for this problem (using the derivatives of the given utility function, which you do not need to verify) is

$$\theta n^{1/\psi} \cdot c = S \cdot w.$$

Solving this expression for ***n*** gives the **labor supply function**

$$n = \left[\frac{S \cdot w}{\theta c}\right]^{\psi}.$$

Finally, taking the natural logarithm of both sides of this expression allows us to re-express the latter expression in logarithmic form:

$$\ln n = \psi \ln w + \psi \ln S - \psi \ln c - \psi \ln \theta$$

(which does not require verification).

In the analysis to follow, you will start from either the logarithmic form of the labor supply function (the latter equation) or the exact form of the labor supply function (the former equation).

Problem 3 continued

Recall from basic microeconomics that the **elasticity** of a variable *x* with respect to another variable *y* is defined as the percentage change in *x* induced by a one-percent change in *y*. As you studied in basic microeconomics, elasticities are especially useful measures of the **sensitivity** of one variable to another because they do not depend on the units of measurement of either variable.

A convenient method for computing an elasticity (which you can take to be true here without proof) is that the elasticity of one variable (say, *x*) with respect to another variable (say, *y*) is equal to the **first derivative of the natural log of *x* with respect to the natural log of *y*.** Read well this statement: it states that the elasticity of the variable *x* with respect to *y* can be computed as $\frac{\partial \ln x}{\partial \ln y}$.

a. **(4 points)** Starting from the logarithmic form of the labor supply condition provided above, compute the elasticity of *n* with respect to the real wage *w*. The resulting expression is the **elasticity of labor supply with respect to the real wage** for the given utility function. Clearly present the steps and logic of your work.

Problem 3 continued

President Obama, his economic team, and many (though certainly not all) members of Congress have implemented a variety of tax measures intended to boost labor-market activity. Along the consumption-labor dimension, what this means is that decreases in the labor-income tax rate t have been put into effect, which equivalently means that increases in the take-home rate S have been put into effect.

b. (4 points) Starting once again from the logarithmic form of the labor supply condition provided above, compute the elasticity of n with respect to the take-home rate S. The resulting expression is the typical measurement of the **elasticity of labor supply with respect to the take-home rate** for the given utility function. Clearly present the steps and logic of your work.

c. (4 points) Based on the result in part b, would a decrease in t be expected to increase labor supply, decrease labor supply, or leave labor supply unchanged? Clearly present the steps and logic of your work and conclusion.

Problem 3 continued

d. **(8 points)** Now start from the exact form of the labor supply condition provided above. Moreover, consider it **in conjunction with the budget constraint** (that is, consider the pair of equations, not just the exact labor supply condition in isolation). Based on this pair of conditions, derive the solution for the optimal choice of n.

e. **(4 points)** Based on the result in part d, would a decrease in t be expected to increase labor supply, decrease labor supply, or leave labor supply unchanged? Clearly present the steps and logic of your work and conclusion.

Problem 3 continued

f. **(6 points – Harder)** Compare your conclusions in part c and part e about how a change in tax policy may affect the quantity of labor supply in the economy. Are the conclusions the same? If so, explain briefly the economics of why; if not, explain briefly the economics of why not; if it is impossible to tell, explain briefly the economics of why a comparison cannot be made.

Problem 4: Financing Constraints and Labor Demand (20 points). In our class discussion about the way in which financing constraints affect firms' profit maximization decisions, we focused on the effects on firms' physical capital investment. In reality, most firms spend twice as much on their wage costs (i.e., their labor costs) than on their physical investment costs. (That is, for most firms, roughly two-thirds of their total costs are wages and salaries, while roughly one-third of their total costs are devoted to maintaining or expanding their physical capital.)

For many firms, payment of wages must be made **before** the receipt of revenues within any given period. (For example, imagine a firm that has to pay its employees to build a computer; the revenues from the sale of this computer typically don't arrive for many weeks or months later because of inherent lags in the shipping process, the retail process, etc.) For this reason, firms typically need to borrow to pay for their payroll costs.[2] But, because of asymmetric information problems, lenders typically require that the firm put up some financial collateral to secure loans for this purpose.

Here, you will analyze the consequences of financing constraints on firms' wage payments using a variation of the accelerator framework we studied in class.

For simplicity, suppose that the representative firm, which operates in a two-period economy, must borrow in order to finance only period-1 wage costs; for some unspecified reason, suppose that period-2 wage costs are not subject to a financing constraint.

As in our study of the accelerator framework in class, the representative firm's two-period discounted profit function is

$$P_1 f(k_1,n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1$$
$$+\frac{P_2 f(k_2,n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

and suppose now the financing constraint that is relevant for firm profit-maximization is

$$P_1 w_1 n_1 = S_1 a_1 .$$

The notation is as always: P denotes the nominal price of the output the firm produces and sells; S denotes the nominal price of stock; D denotes the nominal dividend paid by each unit of stock; n denotes the quantity of labor the firm hires; w is the **real** wage; a_0, a_1, and a_2 are, respectively, the firm's holdings of stock at the end of period 0, period 1, and period 2; k_1, k_2, and k_3 are, respectively, the firm's ownership of physical capital at the end of period 0, period 1, and period 2; i denotes the nominal interest rate between period 1 and period 2; and the production function is denoted by $f(.)$. Also as usual, subscripts on variables denote the time period of reference for that variable. Finally, because this is a two-period framework, we know $a_2 = 0$ and $k_3 = 0$. **(OVER)**

[2] The commercial paper market, about which much has been discussed in the news media in the past couple of years, is one type of channel for such firm financing needs.

Problem 4 continued

a. **(4 points)** Based on the information provided, construct the Lagrangian for the firm's profit maximization problem. Define any new notation you introduce.

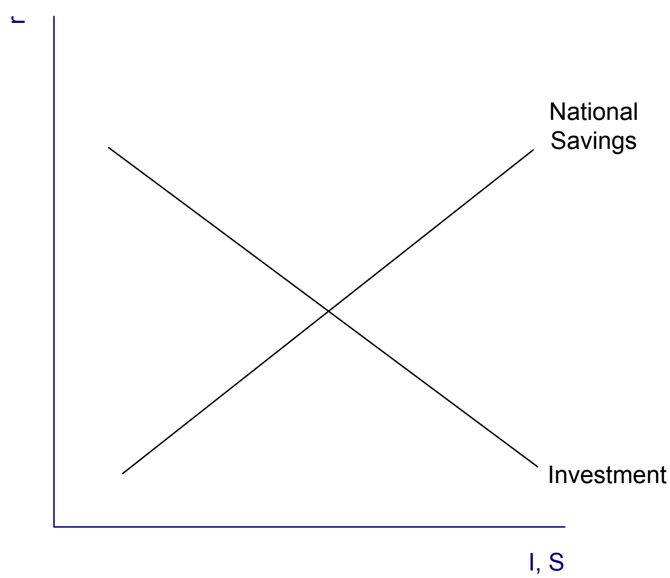
b. **(4 points)** Based on the Lagrangian you constructed in part a, compute the first-order conditions with respect to k_2 and a_1.

Problem 4 continued

c. **(4 points)** Based on the Lagrangian you constructed in part a, compute the first-order conditions with respect to n_1 and n_2.

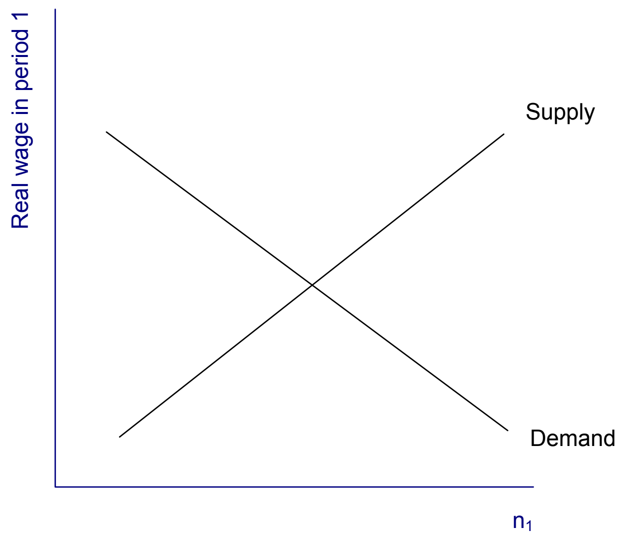
Suppose that, immediately after firm profit maximizing decisions have been made, the real return on STOCK, r^{STOCK}, all of a sudden falls below r, the real return on riskless ("safe") assets. Suppose that before this shock occurred, it was the case that $r = r^{STOCK}$.

d. **(4 points)** Below is a graph of the investment (capital) market in period 1. Does the adverse shock to r^{STOCK} shift either the investment demand and/or the savings supply function? If so, explain how, in what direction, and why.



Problem 4 continued

e. **(4 points)** Below is a graph of the labor market in period 1. Does the adverse shock to r^{STOCK} shift either the labor demand and/or the labor supply function? If so, explain how, in what direction, and why.



Period 1 Labor Market

END OF EXAM