Department of Applied Economics Johns Hopkins University

Economics 602
Macroeconomic Theory and Policy
Final Exam Suggested Solutions
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NAME: ______________________________

The Exam has a total of four (4) problems and pages numbered one (1) through eighteen (18). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use two pages (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 25**
Problem 3	**/ 30**
Problem 4	**/ 20**
TOTAL	**/ 100**

Problem 1: "Hyperbolic Impatience" and Stock Prices (25 points). In this problem you will study a slight extension of the infinite-period economy from Chapter 8. Specifically, suppose the representative consumer has a lifetime utility function given by

$$u(c_t) + \gamma\beta u(c_{t+1}) + \gamma\beta^2 u(c_{t+2}) + \gamma\beta^3 u(c_{t+3}) + ...,$$

in which, as usual, $u(.)$ is the consumer's utility function in any period and β is a number between zero and one that measures the "normal" degree of consumer impatience. **The number γ (the Greek letter "gamma," which is the new feature of the analysis here) is also a number between zero and one, and it measures an "additional" degree of consumer impatience, but one that ONLY applies between period t and period t+1.[1] This latter aspect is reflected in the fact that the factor γ is NOT successively raised to higher and higher powers as the summation grows.**

The rest of the framework is exactly as studied in Chapter 8: a_{t-1} is the representative consumer's holdings of stock at the beginning of period *t,* the nominal price of each unit of stock during period *t* is S_t , and the nominal dividend payment (per unit of stock) during period *t* is D_t . Finally, the representative consumer's consumption during period *t* is c_t and the nominal price of consumption during period *t* is P_t. As usual, analogous notation describes all these variables in periods *t*+1, *t*+2, etc.

The Lagrangian for the representative consumer's utility-maximization problem (starting from the perspective of the beginning of period *t*) is

$$\begin{aligned}
&u(c_t) + \gamma\beta u(c_{t+1}) + \gamma\beta^2 u(c_{t+2}) + \gamma\beta^3 u(c_{t+3}) + ...\\
&+\lambda_t\left[Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t\right]\\
&+\gamma\beta\lambda_{t+1}\left[Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1}c_{t+1} - S_{t+1}a_{t+1}\right]\\
&+\gamma\beta^2\lambda_{t+2}\left[Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2}c_{t+2} - S_{t+2}a_{t+2}\right]\\
&+\gamma\beta^3\lambda_{t+3}\left[Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3}c_{t+3} - S_{t+3}a_{t+3}\right]\\
&+...
\end{aligned}$$

NOTE CAREFULLY WHERE THE "ADDITIONAL" IMPATIENCE FACTOR γ APPEARS IN THE LAGRANGIAN.

(OVER)

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[1] The idea here, which goes under the name "hyperbolic impatience," is that in the "very short run" (i.e., between period t and period t+1), individuals' degree of impatience may be different from their degree of impatience in the "slightly longer short run" (i.e., between period t+1 and period t+2, say). "Hyperbolic impatience" is a phenomenon that routinely recurs in laboratory experiments in experimental economics and psychology, and has many far-reaching economic, financial, policy, and societal implications.

Problem 1 continued

a. **(3 points)** Compute the first-order conditions of the Lagrangian above with respect to **both** a_t **and** a_{t+1}. (**Note:** There is no need to compute first-order conditions with respect to any other variables.)

Solution: The two FOCs are

$$-\lambda_t S_t + \gamma\beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$$
$$-\gamma\beta\lambda_{t+1}S_{t+1} + \gamma\beta^2\lambda_{t+2}(S_{t+2} + D_{t+2}) = 0$$

b. **(4 points)** Using the first-order conditions you computed in part a, construct two distinct stock-pricing equations, one for the price of stock in period *t*, and one for the price of stock in period *t*+1. Your final expressions should be of the form $S_t = ...$ and $S_{t+1} = ...$ (**Note:** It's fine if your expressions here contain Lagrange multipliers in them.)

Solution: Simply rearranging the two FOCs above and canceling the γ term (along with one β term) in the second FOC, we have

$$S_t = \frac{\gamma\beta\lambda_{t+1}}{\lambda_t}(S_{t+1} + D_{t+1})$$
$$S_{t+1} = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}}(S_{t+2} + D_{t+2})$$

For the questions next, observe that the S_t expression and the S_{t+1} expression are subtly, but importantly, different here. They would be identical to each other (other than the fact that the time subscripts are different, but that is as usual) if and only if $\gamma = 1$. If $\gamma < 1$, which is the case of "hyperbolic impatience," then stock prices are determined in a somewhat "different way" in the "very short run" compared to the "longer short run" or "medium run."

Problem 1 continued

For the remainder of this problem, suppose it is known that $D_{t+1} = D_{t+2}$, and that $S_{t+1}=S_{t+2}$, and that $\lambda_t = \lambda_{t+1} = \lambda_{t+2}$.

c. **(5 points).** Does the above information necessarily imply that the economy is in a steady-state? **Briefly and carefully explain why or why not; your response should make clear what the definition of a "steady state" is. (Note:** To address this question, it's possible, though not necessary, that you may need to compute other first-order conditions besides the ones you have already computed above.)

Solution: No, none of these statements **necessarily** implies that the economy is in a steady state, which, recall, means that all real variables become constant and never again change. There are two ways of observing that the above information does not imply the economy is in steady state. First, the above statements are all about **nominal** variables, and in a steady state it can be the case that nominal variables continue fluctuating over time, even though all real variables do not. Another way of arriving at the correct conclusion here is that the statements above only refer to periods t, t+1, and t+2. In a steady-state, (real) variables settle down to constant values **forever,** not just for a few time periods.

d. **(5 points)** Based on the above information and your stock-price expressions from part b, can you conclude that the period-t stock price (S_t) is higher than S_{t+1}, lower than S_{t+1}, equal to S_{t+1}, or is it impossible to determine? **Briefly and carefully explain the economics (i.e., the economic reasoning, not simply the mathematics) of your finding.**

Solution: You are given that nominal stock prices, nominal dividends, and the Lagrange multiplier in period t+1 and t+2 are equal to each other. Let's call these common values $\bar{S}$, $\bar{D}$, and $\bar{\lambda}$ (that is, $\bar{S} = S_{t+1} = S_{t+2}$ $\bar{D} = D_{t+1} = D_{t+2}$, and $\bar{\lambda} = \lambda_{t+1} = \lambda_{t+2}$). Inserting these common values in the period-t+1 stock price equation, we have $\bar{S} = \frac{\beta\bar{\lambda}}{\bar{\lambda}}(\bar{S} + \bar{D})$. Canceling terms, we have that the nominal stock price in period t+1 (and t+2) is $\bar{S} = \beta\bar{S} + \beta\bar{D}$ (which we could of course solve for the stock price as $\bar{S} = \frac{\beta}{1-\beta}\bar{D}$ if we needed to).

Now, using the common values of *S, D*, and the multiplier in the period-t stock price equation gives us $S_t = \gamma\beta(S_{t+1} + D_{t+1}) = \gamma\beta(\bar{S} + \bar{D}) = \gamma(\beta\bar{S} + \beta\bar{D})$. Note that the final term in parentheses is nothing more than $\bar{S}$, hence we have

$$S_t = \gamma\bar{S}.$$

If $\gamma < 1$, then clearly the stock-price in period t is smaller than it is in period t+1 (and period t+2). The economics of this is due to the "hyperbolic impatience" which makes consumers more impatient to purchase consumption in the "very short run" (period t) compared to the "longer short run." All else equal, this means that in the very short run, consumers' do not care to save as much (due to the their extreme impatience in the very short run), which means their demand for saving --- i.e., their demand for stock – is lower. Lower demand for stock means a lower price of stock, all else equal.

Problem 1 continued

Now also suppose that the utility function in every period is $u(c) = \ln c$, and also that the **real interest rate is zero in every period.**

e. **(4 points) Based on the utility function given, the fact that $r = 0$, and the basic setup of the problem described above, construct two marginal rates of substitution (MRS):** the MRS between period-t consumption and period-t+1 consumption, **and** the MRS between period-t+1 consumption and period-t+2 consumption.

Solution: This only requires examining the lifetime utility function (the first line of the Lagrangian above). By definition, the MRS between period t consumption and t+1 consumption is $\frac{u'(c_t)}{\gamma\beta u'(c_{t+1})} = \frac{c_{t+1}}{\gamma\beta c_t}$, and the MRS between period t+1 consumption and t+2 consumption is $\frac{\gamma\beta u'(c_{t+1})}{\gamma\beta^2 u'(c_{t+2})} = \frac{u'(c_{t+1})}{\beta u'(c_{t+2})} = \frac{c_{t+2}}{\beta c_{t+1}}$. Note that the form of the two MRS functions is different: the hyperbolic impatience affects the former MRS, but not the latter MRS.

f. **(4 points – Harder)** Based on the two MRS functions you computed in part e and on the fact that $r = 0$ in every period, determine which of the following two consumption growth rates

$$\frac{c_{t+1}}{c_t} \quad \text{OR} \quad \frac{c_{t+2}}{c_{t+1}}$$

is larger. That is, is the consumption growth rate between period t and period t+1 (the fraction on the left) expected to be larger than, smaller than, or equal to the consumption growth rate between period t+1 and period t+2 (the fraction on the right), or is it impossible to determine? **Carefully explain your logic, and briefly explain the economics (i.e., the economic reasoning, not simply the mathematics) of your finding.**

Solution: The basic consumption-savings optimality condition states that the MRS between two consecutive time periods is equated to (1+r). You are told here that r = 0 always. Based on the two MRS functions constructed above, then, it follows immediately that the consumption growth rate between period t and t+1 is **smaller than** the consumption growth rate between period t+1 and period t+2. This follows because $\gamma < 1$. The economics is similar to above: hyperbolic impatience makes consumers consume “much more” in the very short run (i.e., period t), which means that the growth rate of consumption between period t (already a very high consumption period) and t+1 will be low, compared to the similar comparison one period later.

Problem 2: The Cash-in-Advance (CIA) Framework (25 points). A popular alternative to the money in the utility function (MIU) framework is one in which money holdings directly facilitate transactions – that is, provide a medium of exchange role, one of the basic functions of "money" that the MIU framework captures in only a shortcut form.

Suppose that in any given time period, there are two "types" of consumption goods, "cash goods" and "credit goods." Cash goods, denoted by c_{1t}, are goods whose purchase in period t **requires** money, while credit goods, denoted by c_{2t}, are goods whose purchase in period t does not require money (i.e., they can be bought "on credit"). The market nominal price of each type of good is identical, P_t.

The representative consumer consumes both cash and credit goods. Specifically, suppose the period-t utility function is $u(c_{1t}, c_{2t}, n_t)$, with n_t denoting the individual's **labor** during period t. (As in Chapter 2, suppose that total hours available in any given time period is 168, and the only possible uses of time are labor or leisure.)

The consumer's period-t budget constraint is

$$P_t c_{1t} + P_t c_{2t} + M_t + P_t^b B_t = P_t w_t n_t + M_{t-1} + B_{t-1},$$

Income is earned from labor supply (with w_t denoting the market determined **real** wage in period t, which is taken as given by the individual), and, for simplicity, suppose there are no stock markets (hence one-period riskless bonds and money markets are the only two asset markets). The consumer's bond and money holdings at the start of period *t* are B_{t-1} and M_{t-1}, and at the end of period *t* are B_t and M_t. The individual's budget constraints for period t+1, t+2, … are identical to the above, with the time subscripts appropriately updated. As always, suppose the representative consumer's subjective discount factor between any pair of consecutive time periods is $\beta \in (0,1)$.

In addition to the budget constraint, in each time period the representative consumer **also has a "cash in advance" constraint,**

$$P_t c_{1t} = M_t.$$

The cash-in-advance (CIA) constraint captures the idea that in order to purchase some goods, a certain amount of money (or more generally, "liquidity" such as checkable deposits) has to be held. In principle, the CIA constraint is an inequality constraint (specifically, $P_t c_{1t} \le M_t$), **but in analyzing this problem, you are to assume that it always holds with equality, as written above.** From the standpoint of the analysis you will conduct, because the CIA is technically an inequality constraint, you **may NOT substitute the CIA constraint into the budget constraint.**

Problem 2 continued

The consumer's budget constraints and CIA constraints for period t+1, t+2, … are identical to the above, with the time subscripts appropriately updated.

a. **(4 points)** Set up an appropriate sequential Lagrangian from the perspective of the beginning of period t. Define any new notation you introduce.

Solution: The Lagrangian is

$$\sum_{t=0}^{\infty}\beta^t\left[u(c_{1t},c_{2t},n_t)+\lambda_t\left(P_tw_tn_t+M_{t-1}+B_{t-1}-P_tc_{1t}-P_tc_{2t}-M_t-P_t^bB_t\right)+\mu_t\left(M_t-P_tc_{1t}\right)\right]$$

(note the summation operator) in which λ_t denotes the multiplier on the period-t budget constraint and μ_t denotes the multiplier on the period-t CIA constraint.

b. **(6 points)** Based on the Lagrangian constructed in part a, derive the first-order conditions (FOCs) with respect to period t's choices, c_{1t}, c_{2t}, n_t, B_t, and M_t. Define any new notation you introduce.

Solution: The first-order conditions with respect to $c_{1t},c_{2t},n_t,B_t,M_t$ are, respectively:

$$u_{1t}-\lambda_tP_t-\mu_tP_t=0$$
$$u_{2t}-\lambda_tP_t=0$$
$$u_{3t}+\lambda_tP_tw_t=0$$
$$-\lambda_tP_t^b+\beta\lambda_{t+1}=0$$
$$-\lambda_t+\mu_t+\beta\lambda_{t+1}=0$$

(The shorthand u_{1t} stands for the partial derivative of the utility function with respect to the first argument, and similarly for u_{2t} and u_{3t}.)

Problem 2 continued

c. **(5 points)** Using the FOCs from above, derive the cash good/credit good optimality condition, which should have a final form $\frac{u_1(c_{1t}, c_{2t}, n_t)}{u_2(c_{1t}, c_{2t}, n_t)} = f(i_t)$. **Note that $f(i_t)$ is a function that depends ONLY on the nominal interest rate. You are to determine $f(i_t)$ as part of this problem;** there should be no other variables or parameters at all on the right-hand-side of the cash good/credit good optimality condition you derive. Show clearly the important steps in the algebra.

Solution: Getting to the cash good/credit good optimality condition requires working with the FOCs with respect to cash goods, credit goods, bonds, and money.

The FOC with respect to bonds implies $\beta\lambda_{t+1} = \frac{\lambda_t}{1+i_t}$, which, when substituted into the FOC on money, yields $\frac{\lambda_t}{1+i_t} = \lambda_t - \mu_t$. Dividing through by λ_t,

$$1 - \frac{\mu_t}{\lambda_t} = \frac{1}{1+i_t},$$

or, rewriting slightly,

$$\begin{aligned}\frac{\mu_t P_t}{\lambda_t P_t} &= 1 - \frac{1}{1+i_i} \\ &= \frac{i_t}{1+i_t}\end{aligned}$$

(Obviously, there are many ways to proceed through the algebra here; whichever route struck you as most convenient is fine, the route presented here is just one suggestion.)

Next, from the FOCs on cash consumption and credit consumption, we have $\mu_t P_t = u_{1t} - u_{2t}$, so that $\frac{\mu_t P_t}{\lambda_t P_t} = \frac{u_{1t} - u_{2t}}{u_{2t}} = \frac{u_{1t}}{u_{2t}} - 1$. Inserting this in the last displayed expression above, we have the cash good/credit good optimality condition in the requested form,

$$\frac{u_{1t}}{u_{2t}} = 1 + \frac{i_t}{1+i_t},$$

with the nominal interest rate appearing as the only variable/parameter on the right-hand-side.

Problem 2 continued

d. **(5 points)** Suppose now that the utility function is $u(c_{1t}, c_{2t}, n_t) = \ln c_{1t} + \ln c_{2t} + v(n_t)$, in which $v(n_t)$ is some unspecified function of labor. **Taking into account the fact that the CIA constraint holds with equality at the optimal choice,** derive, based on this utility function and your work above, the **real money demand function,**

$$\frac{M_t}{P_t} = \ldots$$

where the term on the right-hand side is for you to determine. Show clearly the important steps in the algebra. **Also,** qualitatively plot this relationship in a diagram with M/P on the horizontal axis and i on the vertical axis, clearly labeling the two axes.

Solution: The marginal utility functions needed are obviously $u_{1t} = 1/c_{1t}$ and $u_{2t} = 1/c_{2t}$. With the cash-in-advance constraint holding with equality at the optimum, we can equivalently write $u_{1t} = 1/(M_t/P_t)$, and thus can express the cash good/credit good optimality condition as $\frac{u_{1t}}{u_{2t}} = \frac{c_{2t}}{M_t^h / P_t} = 1 + \frac{i_t}{1+i_t}$. Solving for real money balances, we have the real money demand function

$$\frac{M_t}{P_t} = c_{2t}\left[1 + \frac{i_t}{1+i_t}\right]^{-1},$$

which shows that real money demand is decreasing in the nominal interest rate (given that i_t cannot be strictly negative). More technically, compute the derivative of M_t/P_t with respect to i_t, which is always strictly negative. Notice that the money demand function is nothing but the cash good/credit good optimality condition solved for M/P.

A further, simpler, representation of the money demand function is available by recognizing that for this utility function, the cash good/credit good optimality condition is $\frac{c_{2t}}{c_{1t}} = 1 + \frac{i_t}{1+i_t}$; substituting this into the previous representation of the money demand function, we have

$$\frac{M_t}{P_t} = c_{1t},$$

which is, obviously, a restatement of the cash-in-advance constraint holding with equality.

Problem 2 continued

e. **(5 points)** Recall that **in the MIU framework, the consumption-money optimality condition was**

$$\frac{\text{marginal utility of real money holdings}}{\text{marginal utility of consumption}} = \frac{i_t}{1+i_t}.$$

Compare, in terms of economics, this optimality condition to the optimality condition you obtained in part c above for the CIA framework **by briefly commenting on the similarities and differences between the results predicted by the two frameworks. Note:** this does **not** mean restate in words the mathematics; rather, offer two or three thoughts/critiques/etc. on how and why the two frameworks do or not capture the same ideas, how and why the two frameworks perhaps are or are not essentially identical to each other, and so on.

Solution: Each framework captures the idea that money demand is a decreasing function of the nominal interest rate, and each captures the idea of substituting away from cash-intensive activities (purchasing cash goods in the cash/credit model, and enjoying utility from real money balances in the MIU model) and into non-cash activities as the nominal interest rate rises. These are basic ideas in monetary analysis, and each framework articulates them.

An important difference between the two frameworks is that in the cash/credit framework, cash goods **are** viewed as part of the resource frontier of the economy – they are a **good** that must be produced using whatever the economy's production technology is, hence a social planner in this economy would care about "cash goods." In contrast, real money balances in the MIU framework are **not** part of the resource frontier of the economy – a social planner in this economy would not care about real money balances.

This idea manifests itself most clearly in the case of a zero nominal interest rate (i.e., the Friedman Rule). Using the log functional forms given above for ease of exposition, in the cash/credit model $i_t = 0$ implies $\frac{c_{2t}}{c_{1t}} = 1$. In the MIU model, $i_t = 0$ implies $\frac{c_t}{M_t^h / P_t} = 0$, or, a little more informatively, $\frac{M_t^h / P_t}{c_t} = \infty$. In the former case, cash-intensive activity (i.e., purchase of "cash goods") is bounded. In the latter case, cash-intensive activity ("consumption" of money balances) is unbounded: a social planner in this economy would choose to dump an infinite amount of useless pieces of paper into the economy in order to satiate the economy with cash.

Problem 3: Elasticity of Labor Supply and Fiscal Policy (30 points). Consider the static (i.e., one-period) consumption-leisure framework. In quantitative and policy applications that use this framework, a commonly-used utility function is

$$u(c,l) = \ln c - \frac{\theta}{1+1/\psi}(168-l)^{1+1/\psi},$$

in which *c* denotes consumption, *l* denotes the number of hours (in a week) spent in leisure, and ψ and θ (the Greek letters "psi" and "theta," respectively) are **constants** (even though we will not assign any numerical value to them) in the utility function. The representative individual has no control over either ψ or θ, and both $\psi > 0$ and $\theta > 0$.

Labor is measured as $n = 168 - l$, the **real wage** is denoted by *w*, and the labor-income tax rate is denoted by *t*. The **take-home rate** (the fraction of labor income that an individual keeps) can thus be defined as $S = (1-t)$. Finally, expressed in real terms, the representative individual's budget constraint is

$$c = (1-t)wn.$$

Thus, this is **exactly** the consumption-leisure framework studied in Chapter 2, with now a particular functional form for *u*(*c,l*).

The **consumption-leisure optimality condition** for this problem (using the derivatives of the given utility function, which you do not need to verify) is

$$\theta n^{1/\psi} \cdot c = S \cdot w.$$

Solving this expression for ***n*** gives the **labor supply function**

$$n = \left[\frac{S \cdot w}{\theta c}\right]^{\psi}.$$

Finally, taking the natural logarithm of both sides of this expression allows us to re-express the latter expression in logarithmic form:

$$\ln n = \psi \ln w + \psi \ln S - \psi \ln c - \psi \ln \theta$$

(which does not require verification).

In the analysis to follow, you will start from either the logarithmic form of the labor supply function (the latter equation) or the exact form of the labor supply function (the former equation).

Problem 3 continued

Recall from basic microeconomics that the **elasticity** of a variable *x* with respect to another variable *y* is defined as the percentage change in *x* induced by a one-percent change in *y*. As you studied in basic microeconomics, elasticities are especially useful measures of the **sensitivity** of one variable to another because they do not depend on the units of measurement of either variable.

A convenient method for computing an elasticity (which you can take to be true here without proof) is that the elasticity of one variable (say, *x*) with respect to another variable (say, *y*) is equal to the **first derivative of the natural log of *x* with respect to the natural log of *y*.** Read well this statement: it states that the elasticity of the variable *x* with respect to *y* can be computed as $\frac{\partial \ln x}{\partial \ln y}$.

a. **(4 points)** Starting from the logarithmic form of the labor supply condition provided above, compute the elasticity of *n* with respect to the real wage *w*. The resulting expression is the **elasticity of labor supply with respect to the real wage** for the given utility function. Clearly present the steps and logic of your work.

Solution: The most straightforward way to proceed here was to make the following observation(s): first, imagine calling the entire expression $\ln n$ as x. That is, let's temporarily think that $x = \ln n$. Similarly, imagine calling the entire expression $\ln w$ as y. That is, let's temporarily think that $y = \ln w$. With these temporary re-definitions, we can (temporarily) re-express the logarithmic labor supply condition as

$$x = \psi y + \psi \ln S - \psi \ln c - \psi \ln \theta .$$

Computing the derivative of the natural log of *n* with respect to the natural log of *w* (which, according to the heuristic definition of elasticity given above, ***is*** the elasticity *n* with respect to *w*) now just amounts to computing the derivative of *x* with respect to *y* in the last expression. **The elasticity of labor supply with respect to the real wage is thus simply ψ.**

Problem 3 continued

President Obama, his economic team, and many (though certainly not all) members of Congress have implemented a variety of tax measures intended to boost labor-market activity. Along the consumption-labor dimension, what this means is that decreases in the labor-income tax rate t have been put into effect, which equivalently means that increases in the take-home rate S have been put into effect.

b. **(4 points**) Starting once again from the logarithmic form of the labor supply condition provided above, compute the elasticity of n with respect to the take-home rate S. The resulting expression is the typical measurement of the **elasticity of labor supply with respect to the take-home rate** for the given utility function. Clearly present the steps and logic of your work.

Solution: Proceed in a very similar way as in part a above. This time, let's temporarily rename $x = \ln n$ and $y = \ln S$, which allows us to temporarily re-express the logarithmic labor supply condition as $x = \psi \ln w + \psi y - \psi \ln c - \psi \ln \theta$. Computing the derivative of the natural log of n with respect to the natural log of S (which, according to the heuristic definition of elasticity given above, ***is*** the elasticity n with respect to S) now just amounts to computing the derivative of x with respect to y in the last expression. **The elasticity of labor supply with respect to the take-home rate is thus simply** ψ (i.e., exactly the same as in part a)**.**

c. **(4 points)** Based on the result in part b, would a decrease in t be expected to increase labor supply, decrease labor supply, or leave labor supply unchanged? Clearly present the steps and logic of your work and conclusion.

Solution: A decrease in the tax rate t amounts to an equal (in size) increase in the take-home rate S. We just computed in part b that an increase in the take-home rate leads to an increase in labor supply. Which is the final conclusion we need here.

Problem 3 continued

d. **(8 points)** Now start from the exact form of the labor supply condition provided above. Moreover, consider it **in conjunction with the budget constraint** (that is, consider the pair of equations, not just the exact labor supply condition in isolation). Based on this pair of conditions, derive the solution for the optimal choice of *n*.

Solution: Now we must bring the exact form of the labor supply condition together with the budget constraint. From the budget constraint, we can isolate the consumption term, $c = Swn$. Then insert this in the exact form of the labor supply function, which gives

$$n = \left[\frac{S \cdot w}{\theta \cdot S \cdot w \cdot n}\right]^{\psi} = \left[\frac{1}{\theta n}\right]^{\psi}.$$

For the purposes of part e below, we need not proceed further, but because you are asked to actually solve for $n = \ldots$, there are three more algebraic steps needed (actually only two steps if you combined the first two steps presented next). First, pull the *n* on the right-hand side out of the square brackets: with the use of the usual rules of exponents, this gives $n = \left[\frac{1}{\theta}\right]^{\psi} n^{-\psi}$.

Second, multiply both sides by n^{ψ}, which gives $n^{1+\psi} = \left[\frac{1}{\theta}\right]^{\psi}$. Then raise both sides to the power $1/(1+\psi)$, which gives the final solution requested: $n = \left[\frac{1}{\theta}\right]^{\frac{\psi}{1+\psi}}$.

e. **(4 points)** Based on the result in part d, would a decrease in *t* be expected to increase labor supply, decrease labor supply, or leave labor supply unchanged? Clearly present the steps and logic of your work and conclusion.

Solution: It is clear from the final (or even intermediate) expression in part d that *n* **does not depend at all on** taxes! This is clear because this expression does not contain *S* in it. Thus, a change in taxes would not at all affect this "version" of labor supply.

To understand why (though you did not need to conduct this part of the analysis), recall how we constructed what we called the "labor supply function" in Chapter 2: we combined the consumption-leisure optimality condition with the budget constraint, and traced out combinations of wages and (optimal choices of) leisure (labor). Here in part e, doing the same exact procedure shows us that labor supply is independent of taxes – moreover, it is independent of real wages, as well.

Problem 3 continued

f. **(6 points – Harder)** Compare your conclusions in part c and part e about how a change in tax policy may affect the quantity of labor supply in the economy. Are the conclusions the same? If so, explain briefly the economics of why; if not, explain briefly the economics of why not; if it is impossible to tell, explain briefly the economics of why a comparison cannot be made.

Solution: The conclusions in parts c and e are obviously different. The difference is alluded to in the solution to part e: in part e, we are looking at the **full, "macro"** optimal solution (the combination of the consumption-leisure optimality condition and the budget constraint), whereas in part d, we are only considering, intuitively speaking, a "slope" or "micro" argument, rather than both a "slope" and a "level."

In terms of more formal economics, the analysis in part c is tantamount to analyzing the effects of policy on **just** the labor market (why? – because the analysis there treats consumption as a constant). The analysis in part e instead is tantamount to analyzing **jointly** the effects of policy on labor markets **and** goods markets. To the extent that there are feedback effects between the two markets, there is no reason to think the answers from the analyses must be the same. Loosely speaking, we could think of the analysis in part c as a "microeconomic" analysis and the analysis in part e as a "macroeconomic" analysis. What this implies is that one way (perhaps the most important way) to understand the difference between "microeconomic" analysis and "macroeconomic" analysis is that the latter routinely considers feedback effects across markets, whereas the former usually does not.

Problem 4: Financing Constraints and Labor Demand (20 points). In our class discussion about the way in which financing constraints affect firms' profit maximization decisions, we focused on the effects on firms' physical capital investment. In reality, most firms spend twice as much on their wage costs (i.e., their labor costs) than on their physical investment costs. (That is, for most firms, roughly two-thirds of their total costs are wages and salaries, while roughly one-third of their total costs are devoted to maintaining or expanding their physical capital.)

For many firms, payment of wages must be made **before** the receipt of revenues within any given period. (For example, imagine a firm that has to pay its employees to build a computer; the revenues from the sale of this computer typically don't arrive for many weeks or months later because of inherent lags in the shipping process, the retail process, etc.) For this reason, firms typically need to borrow to pay for their payroll costs.[2] But, because of asymmetric information problems, lenders typically require that the firm put up some financial collateral to secure loans for this purpose.

Here, you will analyze the consequences of financing constraints on firms' wage payments using a variation of the accelerator framework we studied in class.

For simplicity, suppose that the representative firm, which operates in a two-period economy, must borrow in order to finance only period-1 wage costs; for some unspecified reason, suppose that period-2 wage costs are not subject to a financing constraint.

As in our study of the accelerator framework in class, the representative firm's two-period discounted profit function is

$$P_1 f(k_1,n_1) + P_1 k_1 + (S_1+D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1$$
$$+\frac{P_2 f(k_2,n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2+D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

and suppose now the financing constraint that is relevant for firm profit-maximization is

$$P_1 w_1 n_1 = S_1 a_1 .$$

The notation is as always: P denotes the nominal price of the output the firm produces and sells; S denotes the nominal price of stock; D denotes the nominal dividend paid by each unit of stock; n denotes the quantity of labor the firm hires; w is the **real** wage; a_0, a_1, and a_2 are, respectively, the firm's holdings of stock at the end of period 0, period 1, and period 2; k_1, k_2, and k_3 are, respectively, the firm's ownership of physical capital at the end of period 0, period 1, and period 2; i denotes the nominal interest rate between period 1 and period 2; and the production function is denoted by $f(.)$. Also as usual, subscripts on variables denote the time period of reference for that variable. Finally, because this is a two-period framework, we know $a_2 = 0$ and $k_3 = 0$. **(OVER)**

[2] The commercial paper market, about which much has been discussed in the news media in the past couple of years, is one type of channel for such firm financing needs.

Problem 4 continued

a. **(4 points)** Based on the information provided, construct the Lagrangian for the firm's profit maximization problem. Define any new notation you introduce.

Solution: The Lagrangian for the firm's profit maximization problem is

$$
\begin{aligned}
&P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 \\
&+ \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i} \\
&+ \lambda\left[S_1 a_1 - P_1 w_1 n_1\right]
\end{aligned}
$$

in which λ denotes the Lagrange multiplier on the financing constraint.

b. **(4 points)** Based on the Lagrangian you constructed in part a, compute the first-order conditions with respect to k_2 and a_1.

Solution: The first-order conditions are simply:

$$
\begin{aligned}
&-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} = 0 \\
&-S_1 + \frac{S_2 + D_2}{1+i} + \lambda S_1 = 0
\end{aligned}
$$

Problem 4 continued

c. **(4 points)** Based on the Lagrangian you constructed in part a, compute the first-order conditions with respect to n_1 and n_2.

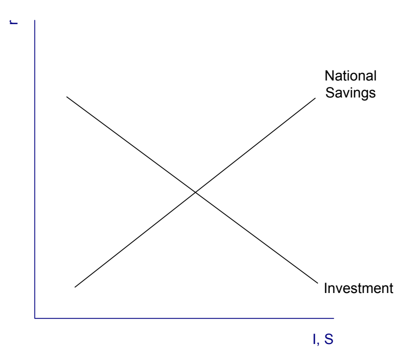
Solution: The first-order conditions are simply:

$$P_1 f_n(k_1, n_1) - P_1 w_1 - \lambda P_1 w_1 = 0$$
$$\frac{P_2 f_n(k_2, n_2)}{1+i} - \frac{P_2 w_2}{1+i} = 0$$

Suppose that, immediately after firm profit maximizing decisions have been made, the real return on STOCK, r^{STOCK}, all of a sudden falls below r, the real return on riskless ("safe") assets. Suppose that before this shock occurred, it was the case that $r = r^{STOCK}$.

d. **(4 points)** Below is a graph of the investment (capital) market in period 1. Does the adverse shock to r^{STOCK} shift either the investment demand and/or the savings supply function? If so, explain how, in what direction, and why.

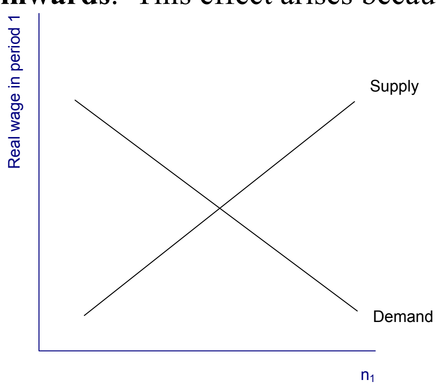
Solution: The investment demand function is unaffected by the financing constraint (see the FOC on $k1$ above), hence exogenous changes in *rSTOCK* have no effect on capital demand/investment demand. Furthermore, because nothing is said about whether financing frictions impinge on the savings supply side of the economy (i.e., on consumers' consumption-savings decisions), there is no basis for asserting any shift of the savings function. Hence, there is no direct effect on the market for physical capital.



Problem 4 continued

e. **(4 points)** Below is a graph of the labor market in period 1. Does the adverse shock to r^{STOCK} shift either the labor demand and/or the labor supply function? If so, explain how, in what direction, and why.

Solution: No shift in labor supply because, as above, no statements are made about whether financing frictions affect consumers' behavior (which is what would be required for a shift of the labor supply function). A fall in *rSTOCK* will cause a RISE in λ, hence for a given level of $w1$, the "effective" marginal product of period-1 labor FALLS, hence the labor demand curve shifts **inwards**. This effect arises because λ directly appears in the period-1 FOC on labor above.



Period 1 Labor Market

END OF EXAM