Department of Applied Economics Johns Hopkins University

Economics 602
Macroeconomic Theory and Policy
Midterm Exam
Professor Sanjay Chugh
Spring 2011
March 7, 2011

NAME: ____

The Exam has a total of four (4) problems and pages numbered one (1) through twelve (12). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use one page (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 25**
Problem 3	**/ 25**
Problem 4	**/ 25**
TOTAL	**/ 100**

Problem 1: Government Sovereignty and the Consequences of Sanctions (25 points). Consider the two-period model of government, with g_1 and g_2 denoting real government spending in periods one and two, and t_1 and t_2 denoting real lump-sum taxes collected by the government in periods one and two.

In class, we discussed the idea that consideration of the government's "utility" function likely involves more than simple economic considerations. Nonetheless, one can study what a government would choose to do if it had some particular some utility function.

Suppose the government's **lifetime** utility function is

$$g_1 - t_1$$

That is, the government **only** cares (in terms of utils) about period one government spending net of tax collections. **However,** due to political considerations, there is an upper limit of 100 on how large a fiscal **surplus** can be run in period two.

The government's lifetime budget constraint is

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0,$$

with r denoting the real interest rate. For simplicity, **suppose throughout this problem that r = 0.** The government's real asset position at the start of period one is b_0, at the end of period one is b_1, and (as usual in the two-period analysis of the government) at the end of period two is $b_2 = 0$.

Suppose that the government begins period one with a negative asset position – that is, suppose $b_0 < 0$.

a. **(3 points)** If $b_0 < 0$, is the government in debt at the beginning of period one? Or is it impossible to determine? Justify/explain **in no more than two phrases/sentences.**

Problem 1 continued

b. **(6 points) Suppose the government can possibly choose to reset b_0 to zero**. That is, by sovereign right of being a government, suppose it can simply "announce" that $b_0 = 0$ even though, absent any such announcement, $b_0 < 0$. Would resetting b_0 to zero possibly allow the government to reach higher lifetime utility? Or would it necessarily decrease the lifetime utility the government could reach? Or would it leave the lifetime utility the government could reach unchanged? Or is it impossible to determine? **Briefly, but thoroughly,** justify/explain.

c. **(11 points)** Suppose that the government can not only possibly choose to reset b_0 to zero (as in part b above), but it **could also choose to reset b_0 to a strictly positive value** (that is, it could choose to set some $b_0 > 0$). **However, if it does set b_0 to a strictly positive value,** the rest of the world imposes "sanctions" on this country's government, which the government is fully aware of. These sanctions cause two things to happen:

   i. Any positive b_0 that the government decides it has are removed by the sanctions; that is, the sanctions cause b_0 to fall back to exactly zero.
   ii. The world's financial markets prohibit this particular government from borrowing at all during period one.

   Taking into account the consequences of the sanctions, would resetting b_0 to a strictly positive value possibly allow the government to reach higher lifetime utility? Or would it necessarily decrease the lifetime utility the government could reach? Or would it leave the lifetime utility the government could reach unchanged? Or is it impossible to determine? **In answering this question, the policy choice of comparison should be the utility consequences of resetting b_0 to zero that was analyzed in part b. Briefly, but thoroughly,** justify/explain

   (OVER)

Problem 1c continued (more work space)

Problem 1 continued

d. **(5 points)** If the goal of the government is to maximize its lifetime utility, answer two related questions:

   i. What should it choose to do regarding b_0? (i.e., should it leave the $b_0 < 0$ as is; should it choose to reset b_0 to zero (as in part b); or should it choose to reset b_0 to a strictly positive value (as in part c))?
   ii. What value for $g_1 - t_1$ should it set in period one?

   (Note: you are to answer BOTH of these questions, and keep in mind the setup of the question described above.) Briefly, but thoroughly, justify/explain.

Problem 2: The Long Run Real Interest Rate (25 points). In this problem, you will analyze the steady state of an infinite-period consumer analysis in which a "credit crunch" is occurring. Specifically, consider a **real** (and simplified) version of the infinite-period consumer framework in which, in each period of time, a budget constraint affects the consumer's optimization, **and a credit restriction also affects the consumer's optimization.** In period *t*, **the credit restriction has the form**

$$c_t = y_t + (1+r_t)a_{t-1}$$

(hence the restriction in period *t*+1 is $c_{t+1} = y_{t+1} + (1+r_{t+1})a_t$, in period *t*+2 is $c_{t+2} = y_{t+2} + (1+r_{t+2})a_{t+1}$, etc.) The consumer's budget constraint in period *t* is

$$c_t + a_t - a_{t-1} = y_t + r_t a_{t-1}$$

(hence the restriction in period *t*+1 is $c_{t+1} + a_{t+1} - a_t = y_{t+1} + r_{t+1}a_t$, in period *t*+2 is $c_{t+2} + a_{t+2} - a_{t+1} = y_{t+2} + r_{t+2}a_{t+1}$, etc.). In the above, the notation for period *t* is the following: c_t denotes consumption in period *t*, r_t denotes the **real** interest rate between period *t*-1 and *t*, a_{t-1} denotes the quantity of assets held at the beginning of period *t*, a_t denotes the quantity of assets held at the end of period *t*, and y_t is the consumer's real income during period *t*. (Similar notation with updated time subscripts describes prices and quantities beyond period *t*.)

Denote by $\beta \in (0,1)$ the subjective discount factor, by $u(c_t)$ the utility function in period *t*, by λ_t the Lagrange multiplier on the period *t* budget constraint, and by ϕ_t (the Greek letter "phi") the Lagrange multiplier on the period *t* credit restriction. (Similar notation with updated time subscripts describes prices, quantities, and multipliers beyond period *t*.)

The Lagrangian of the consumer lifetime utility maximization problem is

$$
\begin{aligned}
&u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \ldots.\\
&+\lambda_t\left[y_t + r_t a_{t-1} - c_t - (a_t - a_{t-1})\right] + \phi_t\left[y_t + (1+r_t)a_{t-1} - c_t\right]\\
&+\beta\lambda_{t+1}\left[y_{t+1} + r_{t+1}a_t - c_{t+1} - (a_{t+1} - a_t)\right] + \beta\phi_{t+1}\left[y_{t+1} + (1+r_{t+1})a_t - c_{t+1}\right]\\
&+\beta^2\lambda_{t+2}\left[y_{t+2} + r_{t+2}a_{t+1} - c_{t+2} - (a_{t+2} - a_{t+1})\right] + \beta^2\phi_{t+2}\left[y_{t+2} + (1+r_{t+2})a_{t+1} - c_{t+2}\right]\\
&+\beta^3\lambda_{t+3}\left[y_{t+3} + r_{t+3}a_{t+2} - c_{t+3} - (a_{t+3} - a_{t+2})\right] + \beta^3\phi_{t+3}\left[y_{t+3} + (1+r_{t+3})a_{t+2} - c_{t+3}\right] + \ldots.
\end{aligned}
$$

Problem 2 continued

a. **(6 points)** Based on the Lagrangian as written above, construct the first-order conditions with respect to c_t, c_{t+1}, and a_t.

b. **(4 points)** In no more than two brief sentences/phrases, describe/define (in general terms, not necessarily just for this problem) an **economic steady state.**

Problem 2 continued

c. **(9 points)** Use **just** the first-order condition on a_t you obtained in part a above to answer the following: in the steady state, does the conclusion $\frac{1}{\beta} = 1 + r$ hold? Or is it impossible to determine? Carefully develop the logic that leads to your conclusion, including showing any key mathematical steps. **Also,** briefly, but thoroughly, explain the **economic interpretation** of your conclusion (i.e., something beyond what is simply apparent from the mathematics).

Problem 2 continued

d. **(6 points)** **Suppose the consumer begins period *t* with zero assets (i.e., $a_{t-1} = 0$).** Also suppose the credit restriction holds with equality in every period. Is the consumer's **savings** positive, negative, or zero in the steady state? Or is it impossible to determine? In answering this question, also briefly define the economic concept of "savings."

Problem 3: Two-Period Economy (25 points). Consider a two-period economy (with no government and hence no taxes), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is $u(c_1, c_2) = \ln c_1 + c_2$, where $\ln$ stands for the natural logarithm (that is not a typo – it is only c_1 that is inside a ln(.) function, c_2 is **not** inside a ln(.) function).

Suppose the following numerical values: the **nominal** interest rate is $i = 0.02$, the nominal price of period-1 consumption is $P_1 = 100$, the nominal price of period-2 consumption is $P_2 = 102$, and the consumer begins period 1 with zero net assets.

a. **(3 points)** Is it possible to numerically compute the **real** interest rate (r) between period one and period two? If so, compute it; if not, explain why not.

b. **(14 points)** Set up a **sequential** Lagrangian formulation of the consumer's problem, in order to answer the following: i) is it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not. ii) is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not.

Problem 3b continued (if you need more space)

c. **(8 points)** The rate of consumption growth between period 1 and period 2 is defined as $\frac{c_2}{c_1} - 1$ (completely analogous to how we have defined, say, the rate of growth of prices between period 1 and period 2). Using **only** the consumption-savings optimality condition for the **given** utility function, **briefly** describe/discuss **(rambling essays will not be rewarded)** whether the real interest rate is **positively related to, negatively related to, or not at all related to the rate of consumption growth between period one and period two.** (**Note:** No mathematics are especially required for this problem; also note this part can be fully completed even if you were unable to get all the way through part b).

Problem 4: European and U.S. Consumption-Leisure Choices (25 points). Europeans work fewer hours than Americans. There are likely very many possible reasons for this, and indeed in reality this fact arises from a combination of many reasons. In this question, you will consider two reasons using the simple (one-period) consumption-leisure model.

a. **(12 points)** Suppose that both the utility functions and pre-tax real wages W/P of American and European individuals are identical. However, the labor income tax rate in Europe is higher than in America. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

Problem 4 continued

b. **(13 points)** Suppose that both the pre-tax real wages W / P and the labor tax rates imposed on American and European individuals are identical. However, the utility function $u^{AMER}(c,l)$ of Americans differs from that of Europeans $u^{EUR}(c,l)$. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

END OF EXAM