Department of Applied Economics | Johns Hopkins University

Economics 602

Macroeconomic Theory and Policy

Midterm Exam – Suggested Solutions

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NAME: ______________________________

Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use one page (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 25**
Problem 3	**/ 25**
Problem 4	**/ 25**
TOTAL	**/ 100**

Problem 1: Government Sovereignty and the Consequences of Sanctions (25 points). Consider the two-period model of government, with g_1 and g_2 denoting real government spending in periods one and two, and t_1 and t_2 denoting real lump-sum taxes collected by the government in periods one and two.

In class, we discussed the idea that consideration of the government's "utility" function likely involves more than simple economic considerations. Nonetheless, one can study what a government would choose to do if it had some particular utility function.

Suppose the government's **lifetime** utility function is

$$g_1 - t_1$$

That is, the government **only** cares (in terms of utils) about period one government spending net of tax collections. **However,** due to political considerations, there is an upper limit of 100 on how large a fiscal **surplus** can be run in period two.

The government's lifetime budget constraint is

$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} + (1+r)b_0 ,$$

with r denoting the real interest rate. For simplicity, **suppose throughout this problem that r = 0.** The government's real asset position at the start of period one is b_0, at the end of period one is b_1, and (as usual in the two-period analysis of the government) at the end of period two is $b_2 = 0$.

Suppose that the government begins period one with a negative asset position – that is, suppose $b_0 < 0$.

a. **(3 points)** If $b_0 < 0$, is the government in debt at the beginning of period one? Or is it impossible to determine? Justify/explain **in no more than two phrases/sentences.**

Solution: By definition, b_{t-1} is the government's net **asset** position at the start of any period t. Thus, a negative value means a net **debt** position; the government is thus in debt at the beginning of period one.

Problem 1 continued

b. **(6 points) Suppose the government can possibly choose to reset b_0 to zero**. That is, by sovereign right of being a government, suppose it can simply "announce" that $b_0 = 0$ even though, absent any such announcement, $b_0 < 0$. Would resetting b_0 to zero possibly allow the government to reach higher lifetime utility? Or would it necessarily decrease the lifetime utility the government could reach? Or would it leave the lifetime utility the government could reach unchanged? Or is it impossible to determine? **Briefly, but thoroughly,** justify/explain.

Solution: To address this question (as well as part c), it is helpful to rewrite the lifetime budget constraint given above to

$$g_1 - t_1 = \frac{t_2 - g_2}{1+r} + (1+r)b_0 .$$

What is useful about this rewriting of the budget constraint is that the term $g_1 - t_1$ (which is the government's lifetime utility function) appears on the left hand side. You are given that $r = 0$ and that **$t_2 - g_2$ (i.e., the fiscal surplus in period two) cannot be larger than 100.** With a strictly negative b_0, the right hand side is necessarily strictly smaller than 100, which in turn implies that $g_1 - t_1$ is necessarily strictly smaller than 100. If the government can reset b_0 to zero, then the right hand side could be as large as 100, which in turn implies that $g_1 - t_1$ could be as large as 100. Thus, this policy choice (which is a government "default" on its existing debt obligations) allows the government to achieve higher lifetime utility.

c. **(11 points)** Suppose that the government can not only possibly choose to reset b_0 to zero (as in part b above), but it **could also choose to reset b_0 to a strictly positive value** (that is, it could choose to set some $b_0 > 0$). **However, if it does set b_0 to a strictly positive value,** the rest of the world imposes "sanctions" on this country's government, which the government is fully aware of. These sanctions cause two things to happen:

   i. Any positive b_0 that the government decides it has are removed by the sanctions; that is, the sanctions cause b_0 to fall back to exactly zero.
   ii. The world's financial markets prohibit this particular government from borrowing at all during period one.

   Taking into account the consequences of the sanctions, would resetting b_0 to a strictly positive value possibly allow the government to reach higher lifetime utility? Or would it necessarily decrease the lifetime utility the government could reach? Or would it leave the lifetime utility the government could reach unchanged? Or is it impossible to determine? **In answering this question, the policy choice of comparison should be the utility consequences of resetting b_0 to zero that was analyzed in part b. Briefly, but thoroughly,** justify/explain

Solution: The analysis in part b concluded that if the government "chose" to move to a higher level of b_0 (i.e., moving from strictly negative b_0 to $b_0 = 0$), it would be able to achieve higher lifetime utility. It may stand to reason then that moving to a strictly positive b_0 would enable it

Problem 1c continued (more work space)

to achieve an even higher utility.

With the sanctions described, however, this is impossible. If the government attempts to set $b_0 > 0$ (which is tantamount to the government "creating assets" for itself), the sanctions lower b_0 down to zero (which can be interpreted as governments in the rest of the world "seizing" the government's newly created "assets"). Moreover, and importantly, the government **cannot** spend more in period one than its tax collections in period 1 as a consequence of the second component of the sanctions.

The latter conclusion follows from inspecting the budget constraint as expressed in part b along with the following argument: with $b_0 = 0$ (due to the first component of the sanctions) and the impossibility of setting $t_2 - g_2$ larger than 100, the government **could** run a fiscal **deficit** in period one of $g_1 - t_1$ of as large as 100. But in order to do so, the government would have to borrow in period one (i.e., be in debt at the end of period one).

The second component of the sanctions prevents the government from borrowing in period one, hence the best the government can do is implement $g_1 - t_1 = 0$ in period one.

Thus, choosing to "create assets" necessarily decreases the lifetime utility the government could achieve, given the nature of sanctions that would be imposed on the country.

Problem 1 continued

d. **(5 points)** If the goal of the government is to maximize its lifetime utility, answer two related questions:

   i. What should it choose to do regarding b_0? (i.e., should it leave the $b_0 < 0$ as is; should it choose to reset b_0 to zero (as in part b); or should it choose to reset b_0 to a strictly positive value (as in part c))?
   ii. What value for $g_1 - t_1$ should it set in period one?

   (Note: you are to answer BOTH of these questions, and keep in mind the setup of the question described above.) Briefly, but thoroughly, justify/explain.

Solution: As implied by the analysis of parts b and c, the government should "default" on its debt and declare $b_0 = 0$, but **not** set $b_0 > 0$. And it should choose to run the largest possible **deficit** it can in period one; given the impossibility of running a fiscal surplus larger than 100 in period two, this means it should implement a fiscal deficit of 100 in period one: $t_1 - g_1 = -100$ (negative).

Problem 2: The Long Run Real Interest Rate (25 points). In this problem, you will analyze the steady state of an infinite-period consumer analysis in which a "credit crunch" is occurring. Specifically, consider a **real** (and simplified) version of the infinite-period consumer framework in which, in each period of time, a budget constraint affects the consumer's optimization, **and a credit restriction also affects the consumer's optimization.** In period *t*, **the credit restriction has the form**

$$c_t = y_t + (1+r_t)a_{t-1}$$

(hence the restriction in period *t*+1 is $c_{t+1} = y_{t+1} + (1+r_{t+1})a_t$, in period *t*+2 is $c_{t+2} = y_{t+2} + (1+r_{t+2})a_{t+1}$, etc.) The consumer's budget constraint in period *t* is

$$c_t + a_t - a_{t-1} = y_t + r_t a_{t-1}$$

(hence the restriction in period *t*+1 is $c_{t+1} + a_{t+1} - a_t = y_{t+1} + r_{t+1}a_t$, in period *t*+2 is $c_{t+2} + a_{t+2} - a_{t+1} = y_{t+2} + r_{t+2}a_{t+1}$, etc.). In the above, the notation for period *t* is the following: c_t denotes consumption in period *t*, r_t denotes the **real** interest rate between period *t*-1 and *t*, a_{t-1} denotes the quantity of assets held at the beginning of period *t*, a_t denotes the quantity of assets held at the end of period *t*, and y_t is the consumer's real income during period *t*. (Similar notation with updated time subscripts describes prices and quantities beyond period *t*.)

Denote by $\beta \in (0,1)$ the subjective discount factor, by $u(c_t)$ the utility function in period *t*, by λ_t the Lagrange multiplier on the period *t* budget constraint, and by ϕ_t (the Greek letter "phi") the Lagrange multiplier on the period *t* credit restriction. (Similar notation with updated time subscripts describes prices, quantities, and multipliers beyond period *t*.)

The Lagrangian of the consumer lifetime utility maximization problem is

$$\begin{aligned}
&u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \beta^3 u(c_{t+3}) + \dots \\
&+\lambda_t \left[y_t + r_t a_{t-1} - c_t - (a_t - a_{t-1}) \right] + \phi_t \left[y_t + (1+r_t)a_{t-1} - c_t \right] \\
&+\beta\lambda_{t+1} \left[y_{t+1} + r_{t+1} a_t - c_{t+1} - (a_{t+1} - a_t) \right] + \beta\phi_{t+1} \left[y_{t+1} + (1+r_{t+1})a_t - c_{t+1} \right] \\
&+\beta^2\lambda_{t+2} \left[y_{t+2} + r_{t+2} a_{t+1} - c_{t+2} - (a_{t+2} - a_{t+1}) \right] + \beta^2\phi_{t+2} \left[y_{t+2} + (1+r_{t+2})a_{t+1} - c_{t+2} \right] \\
&+\beta^3\lambda_{t+3} \left[y_{t+3} + r_{t+3} a_{t+2} - c_{t+3} - (a_{t+3} - a_{t+2}) \right] + \beta^3\phi_{t+3} \left[y_{t+3} + (1+r_{t+3})a_{t+2} - c_{t+3} \right] + \dots
\end{aligned}$$

Problem 2 continued

a. **(6 points)** Based on the Lagrangian as written above, construct the first-order conditions with respect to c_t, c_{t+1}, and a_t.

Solution: The FOCs are:

$$u'(c_t) - \lambda_t - \phi_t = 0$$
$$-\lambda_t + \beta\lambda_{t+1}(1 + r_{t+1}) + \beta\phi_{t+1}(1 + r_{t+1}) = 0$$
$$\beta u'(c_{t+1}) - \beta\lambda_{t+1} - \beta\phi_{t+1} = 0$$

b. **(4 points)** In no more than two brief sentences/phrases, describe/define (in general terms, not necessarily just for this problem) an **economic steady state.**

Solution: An economic steady state is a condition in which all prices and quantities that are measured in **real** terms (i.e., in units of the aggregate consumption basket) become constant (stop fluctuating from one time period to the next).

Problem 2 continued

c. **(9 points)** Use **just** the first-order condition on a_t you obtained in part a above to answer the following: in the steady state, does the conclusion $\frac{1}{\beta} = 1 + r$ hold? Or is it impossible to determine? Carefully develop the logic that leads to your conclusion, including showing any key mathematical steps. **Also,** briefly, but thoroughly, explain the **economic interpretation** of your conclusion (i.e., something beyond what is simply apparent from the mathematics).

Solution: Imposing steady state on the FOC on a_t requires dropping all the time subscripts since every object in the expression **is** a real object. Dropping time subscripts gives us

$$\lambda = \beta\lambda(1+r) + \beta\phi(1+r).$$

Divide this expression by λ and then divide the resulting expression by β, which gives

$$\frac{1}{\beta} = 1 + r + \frac{\phi}{\lambda}(1+r) = (1+r)\left(1 + \frac{\phi}{\lambda}\right)$$

(the term following the second equals sign simply groups terms together; you did not have to group terms this way).

Clearly, if $\phi = 0$, then we get the "usual" steady state relationship $\frac{1}{\beta} = 1 + r$. However, if $\phi \neq 1$, the relationship is not satisfied. Broadly, the reason is that the "usual" relationship is predicated on the view that, at least in the long run (i.e., the steady state), credit restrictions do not affect consumption purchases (even though they may in the short run, i.e., before the steady state is reached).

If credit restrictions do affect consumption purchases in the long run, "how severely" the credit restrictions affect choices (which is captured by the multiplier ϕ) alter the relationship between market returns 1+r and impatience β. (We will study long run distortions imposed on the economy by imperfect competition in financial markets later in the semester.)

Problem 2 continued

d. **(6 points) Suppose the consumer begins period *t* with zero assets (i.e., $a_{t-1} = 0$).** Also suppose the credit restriction holds with equality in every period. Is the consumer's **savings** positive, negative, or zero in the steady state? Or is it impossible to determine? In answering this question, also briefly define the economic concept of "savings."

Solution: To answer this question, start by inspecting the credit restriction and the budget constraint for period *t*. Given zero assets at the beginning of period *t*, combining the period-*t* budget constraint and credit constraint leads to the conclusion

$$a_t = 0.$$

Then repeat this argument for period *t*+1 (i.e., combine the period-*t*+1 budget constraint and credit constraint, now using $a_t = 0$); this leads to the conclusion that

$$a_{t+1} = 0.$$

Clearly, replicating this argument forward leads to the conclusion that asset holdings at the end of every period are zero, including in the steady state.

You are asked about savings (a flow), not asset holdings per se (an accumulation variable). In any given period, savings is defined as the change in asset holdings during the course of that period – clearly, savings equals zero, in every time period, including in the steady state.

Problem 3: Two-Period Economy (25 points). Consider a two-period economy (with no government and hence no taxes), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is $u(c_1, c_2) = \ln c_1 + c_2$, where $\ln$ stands for the natural logarithm (that is not a typo – it is only c_1 that is inside a ln(.) function, c_2 is **not** inside a ln(.) function).

Suppose the following numerical values: the **nominal** interest rate is $i = 0.02$, the nominal price of period-1 consumption is $P_1 = 100$, the nominal price of period-2 consumption is $P_2 = 102$, and the consumer begins period 1 with zero net assets.

a. **(3 points)** Is it possible to numerically compute the **real** interest rate (r) between period one and period two? If so, compute it; if not, explain why not.

Solution: The inflation rate is easily computed as $\pi_2 = \frac{P_2}{P_1} - 1 = \frac{102}{100} - 1 = 0.02$. Then, using the exact Fisher equation, $1 + r = \frac{1+i}{1+\pi_2} = \frac{1.02}{1.02} = 1$, so that $r = 0$.

b. **(14 points)** Set up a **sequential** Lagrangian formulation of the consumer's problem, in order to answer the following: i) is it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not. ii) is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not.

Solution: The sequential Lagrangian for this problem (here cast in real terms, but you could have case it in nominal terms as well) is

$$u(c_1, c_2) + \lambda_1[y_1 - c_1 - a_1] + \lambda_2[y_2 + (1+r)a_1 - c_2],$$

where λ_1 and λ_2 are the multipliers on the period-1 and period-2 budget constraints. The first-order condition with respect to c_1 is $u_1(c_1, c_2) - \lambda_1 = 0$, with respect to c_2 is $u_2(c_1, c_2) - \lambda_2 = 0$, and with respect to a_1 is $-\lambda_1 + \lambda_2(1+r) = 0$. The third FOC allows us to conclude $\lambda_1 = \lambda_2(1+r)$. Substituting this into the FOC on c_1 gives $u_1(c_1, c_2) = \lambda_2(1+r)$. Next, the FOC on c_2 allows us to obtain $\lambda_2 = u_2(c_1, c_2)$. Substituting this into the previous expression gives us $u_1(c_1, c_2) = u_2(c_1, c_2)(1+r)$, or $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = 1 + r$, which of course is the usual consumption-savings optimality condition. Using the given functional form, the consumption-savings optimality condition for this problem can be expressed as $\frac{1/c_1}{1} = 1 + r$, which immediately allows us to conclude $c_1 = \frac{1}{1+r} = \frac{1}{1} = 1$, which completes part i. However, c_2 **cannot** be computed here because you are given no numerical values regarding income, either in present-value or period-by-period form.

Problem 3 continued

c. **(8 points)** The rate of consumption growth between period 1 and period 2 is defined as $\frac{c_2}{c_1} - 1$ (completely analogous to how we have defined, say, the rate of growth of prices between period 1 and period 2). Using **only** the consumption-savings optimality condition for the **given** utility function, **briefly** describe/discuss **(rambling essays will not be rewarded)** whether the real interest rate is **positively related to, negatively related to, or not at all related to the rate of consumption growth between period one and period two.** (**Note:** No mathematics are especially required for this problem; also note this part can be fully completed even if you were unable to get all the way through part b).

Solution: The familiar consumption-savings optimality condition is $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = 1 + r$. As we just saw above, for the given utility function, this becomes $\frac{1/c_1}{1} = 1 + r$, or, rearranging,

$$c_1 = \frac{1}{1+r}.$$

For the consumption-savings optimality condition associated with this particular utility function (which is **quasi-linear** in period-2 consumption), r seems to affect only the period-1 optimal choice of consumption and does **not** affect the growth rate of consumption across periods. Since you were asked to base your analysis on the consumption-savings optimality condition, the conclusion would thus be that r is not at all related to the rate of consumption growth for this utility function, instead affecting only the short-run level of consumption.

However, it is the case that in the full solution to the problem (i.e., using the consumption-savings optimality condition in tandem with the consumer's lifetime budget constraint to solve jointly for both short-run and long-run consumption), c_2 rises when r rises (to see this, substitute the consumption-savings optimality condition into the LBC, and solve for c_2). The fact that c_2 rises when r rises coupled with the result that c_1 falls when r rises means that indeed the consumption growth rate between period 1 and period 2 rises when r rises. You were not required to take the analysis this far since you were asked only to base the analysis on the consumption-savings optimality condition – however (and many answers ran into this difficulty), **if** you decided to take this route you had to take it correctly.

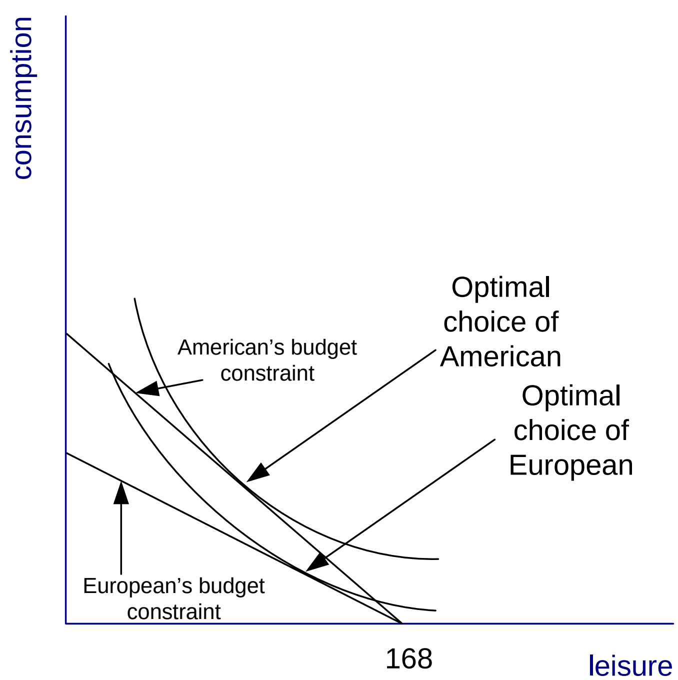
Many answers also simply discussed vaguely the consumption-savings optimality condition to argue something – you were told to base the analysis on the given utility function, so a general analysis did not address the issue.

Finally, note that simply arguing/explaining here that a rise in the real interest rate leads to a fall in period-1 consumption does not address the question – the question is about the **rate of change of consumption between period 1 and period 2,** not about the **level** of consumption in period 1 by itself.

Problem 4: European and U.S. Consumption-Leisure Choices (25 points). Europeans work fewer hours than Americans. There are likely very many possible reasons for this, and indeed in reality this fact arises from a combination of many reasons. In this question, you will consider two reasons using the simple (one-period) consumption-leisure model.

a. **(12 points)** Suppose that both the utility functions and pre-tax real wages W/P of American and European individuals are identical. However, the labor income tax rate in Europe is higher than in America. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

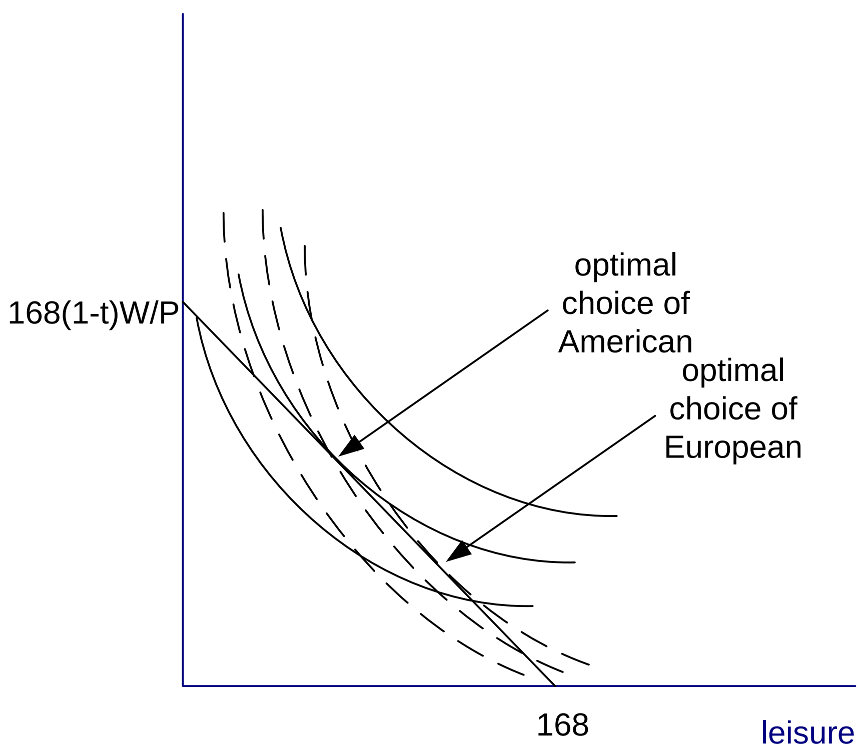
Solution: If Europeans work fewer hours than Americans, then Europeans have more leisure time than Americans, simply because (in our weekly framework) $n+l=168$. Europeans and Americans have identical utility functions, which means that their indifference maps are identical. This means that the difference in hours worked must arise completely from differences in their budget constraints. With a higher labor income tax in Europe, the budget constraint of the European consumer is less steep than the budget constraint of the American, as the diagram below shows (because the slope of the budget constraint is $(1-t)W/P$, and you are given that W/P is the same in the two countries). The diagram shows that the European optimally chooses more leisure (hence less labor) and less consumption than the American. Here, the difference between Europeans and Americans is solely in the relative prices (embodied by the slope of the budget constraint) they face. (For full credit here, you had to somehow make clear that the indifference maps of the representative European and the representative American are identical.)



Problem 4 continued

b. **(13 points)** Suppose that both the pre-tax real wages W/P and the labor tax rates imposed on American and European individuals are identical. However, the utility function $u^{AMER}(c,l)$ of Americans differs from that of Europeans $u^{EUR}(c,l)$. In a **single** carefully-labeled indifference-curve/budget constraint diagram (with consumption on the vertical axis and leisure on the horizontal axis), show how it can be the case that Europeans work fewer hours than Americans. Provide any explanation of your diagram that is needed.

Solution: In this case, the budget constraints of the European consumer and American consumer are identical, so the difference in hours worked must arise completely from differences in their utility functions. Graphically, this means that the two types of consumers have different indifference maps (i.e., a different set of indifference curves). In the diagram below, the budget line is the common budget line of the European and the American. The solid indifference curves are the American's, while the dashed indifference curves are the European's. With steeper indifference curves, the European's optimal choice along the same budget line must occur at a point that features more leisure (hence less labor) and less consumption than the American's optimal choice. Here, the difference between Europeans and Americans is solely in their preferences.



END OF EXAM

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