Department of Applied Economics Johns Hopkins University

Economics 602
Macroeconomic Theory and Policy
Midterm Exam – Suggested Solutions
Professor Sanjay Chugh
Spring 2012

NAME: ____________________

The Exam has a total of four (4) problems and pages numbered one (1) through sixteen (16). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.**

In particular, some of the questions state explicit WORD COUNT limits – respect these limits. Grading of such questions will strictly STOP after that number of words has been reached.

You are to answer all questions in the spaces provided.

You may use **one page (double-sided) of hand-written notes.** You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 30**
Problem 3	**/ 30**
Problem 4	**/ 15**
TOTAL	**/ 100**

Problem 1: Consumption and Savings in the Two-Period Economy (25 points). Consider a two-period economy (with no government), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is

$$u(c_1, c_2) = \ln c_1 + \ln c_2,$$

where $\ln$ stands for the natural logarithm. **All analysis is conducted from the very start of period one.**

a. **(2 points)** What is the **marginal** utility function for consumption goods in period one?

Solution: The MU function for good one is simply $1/c_1$.

b. **(2 points)** What is the **marginal** utility function for consumption goods in period two?

Solution: The MU function for good one is simply $1/c_2$.

c. **(4 points)** Based on the above two answers, what is the marginal rate of substitution (MRS) between period-one consumption and period-two consumption? **In your construction of the MRS, explain in NO MORE THAN 10 WORDS what it is that is being constructed.** (Express your final answer as $\frac{u_1(c_1,c_2)}{u_2(c_1,c_2)}$.)

Solution: The marginal rate of substitution measures how much of one good a representative individual would give up in order to obtain one more unit of the other good. Based on the above answers, the MRS between c_1 and c_2 is

$$MRS(c_1, c_2) = \frac{c_2}{c_1}.$$

Problem 1 continued

d. **(4 points)** One way to think about the **real interest rate** in macroeconomics (in addition to the couple of different interpretations we have already discussed in class) is that it reflects the **rate of consumption growth between two consecutive periods.**

IN 40 WORDS OR LESS: using **ONLY** the consumption-savings optimality condition for the given utility function (i.e., the answer to part c), **briefly** describe/discuss whether the real interest rate is **positively related to, negatively related to, or not at all related to the rate of consumption growth between period one and period two.** For your reference, the definition of the rate of consumption growth rate between period one and period two is $\frac{c_2}{c_1} - 1$ (completely analogous to how we defined in class the rate of growth of prices between period one and period two). (**Note:** No mathematics are especially required for this problem.)

Solution: The familiar condition is $\left(\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = \right) \frac{c_2}{c_1} = 1 + r$.

For the consumption-savings optimality condition associated with this particular utility function, r clearly affects the **growth rate** of consumption between period one and period two.

Problem 1 continued

For the rest of the problem, we will work in purely **real** terms. Suppose the consumer's **present discounted value of ALL lifetime REAL income is 26.** Also suppose the consumer begins period one with zero net assets ($a_0 = 0$). (And as a reminder: note again that the lifetime utility function of the representative consumer is $u(c_1, c_2) = \ln c_1 + \ln c_2$.)

e. **(4 points)** Set up the **sequential** Lagrangian formulation for the representative consumer. If you include any new variables, define them carefully.

Solution: The sequential Lagrangian for this problem (here cast in real terms, but you could have case it in nominal terms as well) is

$$u(c_1, c_2) + \lambda_1[y_1 - c_1 - a_1] + \lambda_2[y_2 + (1+r)a_1 - c_2],$$

where λ_1 and λ_2 are the multipliers on the period one and period two budget constraints.

f. **(9 points)** Based on the **sequential** Lagrangian above, **solve as fully as possible** the consumer's optimal choices in period one and in period two. Specifically, there are **three** questions to address:

i) Is it possible to **numerically** compute the consumer's optimal choice of consumption in period one? If so, compute it; if not, **briefly** explain why not.

ii) Is it possible to **numerically** compute the consumer's optimal choice of consumption in period two? If so, compute it; if not, **briefly** explain why not.

iii) Is it possible to **numerically** compute the consumer's real asset position at the end of period one? If so, compute it; if not, **briefly** explain why not.

Solution: The first-order condition with respect to c_1 (written generally) is $u_1(c_1, c_2) - \lambda_1 = 0$, with respect to c_2 is $u_2(c_1, c_2) - \lambda_2 = 0$, and with respect to a_1 is $-\lambda_1 + \lambda_2(1+r) = 0$. The third FOC allows us to conclude $\lambda_1 = \lambda_2(1+r)$. Substituting this into the FOC on c_1 gives $u_1(c_1, c_2) = \lambda_2(1+r)$.

Problem 1f continued (more work space if needed)

Next, the FOC on c_2 allows us to obtain $\lambda_2 = u_2(c_1, c_2)$. Substituting this into the previous expression gives us $u_1(c_1, c_2) = u_2(c_1, c_2)(1+r)$, or $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = 1+r$, which of course is the usual consumption-savings optimality condition.

Using the given functional form, the consumption-savings optimality condition for this problem can be expressed as $\frac{1/c_1}{1/c_2} = 1+r$, which, when combined with the lifetime budget constraint, immediately gives us that

$$c_1 = 13.$$

However, we can NOT conclude anything about either the optimal choice of c2 or the optimal choice of a1 because we don’t know anything about the real interest rate, *r*.

Problem 2: The Consumption-Leisure Framework (30 points). In this question, you will use the basic (one period) consumption-leisure framework to consider some labor market issues.

Suppose the representative consumer has the following utility function over consumption and **labor,**

$$u(c,l) = \ln c - An,$$

where, as usual, c denotes consumption and n denotes the number of hours of labor the consumer chooses to work. The constant $A > 0$ is outside the control of the individual. (As usual, $\ln(\cdot)$ is the natural log function.)

Suppose the budget constraint (expressed in real, rather than in nominal, terms) the individual faces is $c = (1-t)\cdot w \cdot n$, where t is the labor tax rate, w is the **real** hourly wage rate, and n is the number of hours the individual works.

Recall that in one week there are 168 hours, hence $n + l = 168$ must always be true.

a. **(4 points)** Construct the Lagrangian for the consumer's utility maximization problem, defining any new notation you need to include.

Solution: The Lagrangian is

$$\ln c - An + \lambda\left[(1-t)wn - c\right],$$

in which λ is the Lagrange multiplier.

b. **(4 points)** Based on the Lagrangian in part a, compute the representative consumer's first-order conditions with respect to consumption and with respect to labor.

Solution: The first-order conditions on c and n are

$$\frac{1}{c} - \lambda = 0$$

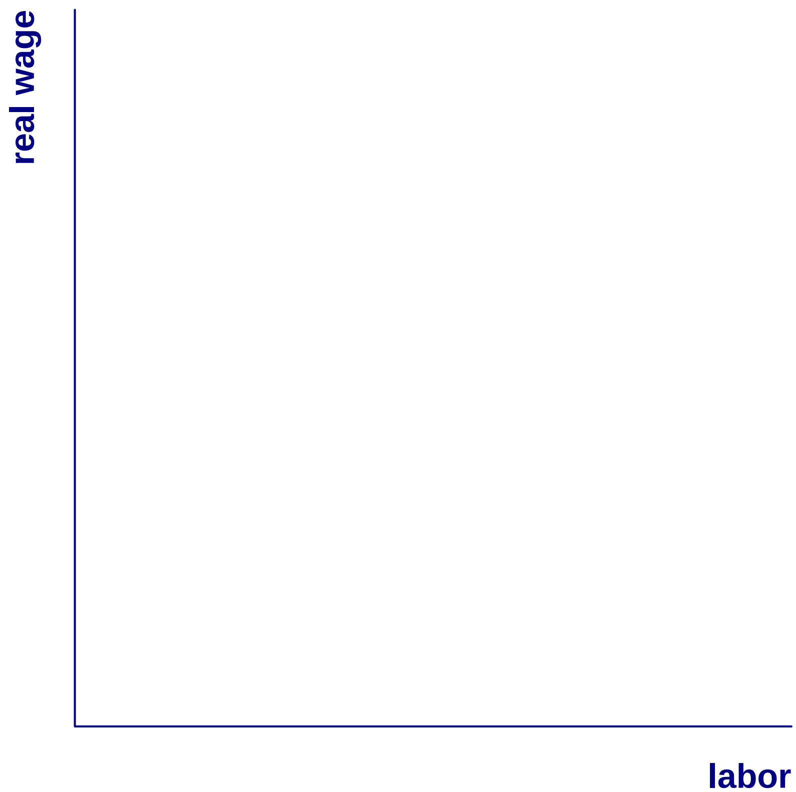
$$-A + \lambda(1-t)w = 0$$

Problem 2 continued

c. **(6 points)** Based on **ONLY** the first-order condition with respect to labor computed in part b, **qualitatively** sketch two things in the diagram below **and briefly address** one question.

First, sketch the general shape of the relationship between w and n (perfectly vertical, perfectly horizontal, upward-sloping, downward-sloping, or impossible to tell). **Second,** sketch how changes in t affect the relationship (shift it outwards, shift it in inwards, or impossible to determine). **And, briefly (in no more than 10 words!)** describe the **economics** of how you obtained your conclusions. (**IMPORTANT NOTE:** In this question, you are **not** to use the first-order condition with respect to consumption nor any other conditions.)

Solution: Using just the FOC on labor above, there is a **perfectly horizontal** labor supply function that emerges in the diagram below. This is because n simply does not appear in the FOC on labor. Second, because t does appear, it causes the labor supply function to shift up or down. This labor supply function is **perfectly elastic** (not shown below….).



Problem 2 continued

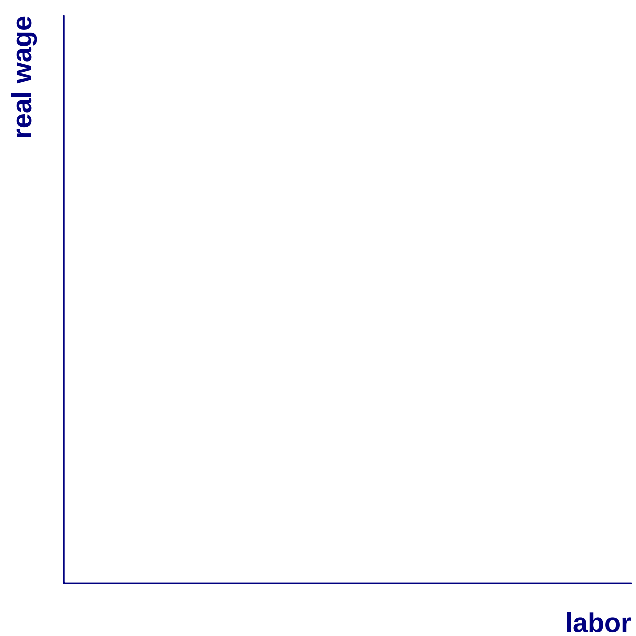
d. **(4 points)** Now based on **both** of the two first-order conditions computed in part b, construct the consumption-leisure optimality condition (which technically in this question is the "consumption-labor" optimality condition, but that is a technical detail).

Solution: Proceeding as usual, the FOC on c gives us $\lambda = \frac{1}{c}$, which when inserted in the FOC on labor, gives us $A = \frac{(1-t)w}{c}$. With an algebraic rearrangement (multiplying through by c), we have the consumption-leisure (more properly, the consumption-labor) optimality condition $Ac = (1-t)w$.

e. **(6 points)** Based on both the "consumption-leisure" optimality condition obtained in part d and on the budget constraint, **qualitatively** sketch two things in the diagram below **and briefly address** one question.

First, sketch the general shape of the relationship between w and n (perfectly vertical, perfectly horizontal, upward-sloping, downward-sloping, or impossible to tell). **Second,** sketch how changes in t affect the relationship (shift it outwards, shift it in inwards, or impossible to determine). **And, briefly (in no more than 10 words!)** describe the **economics** of how you obtained your conclusions.

Solution: From part d above, we have $Ac = (1-t)w$. And the budget constraint is $c = (1-t)wn$. Substituting the latter into the former gives $n = A$ (> 0). The labor supply function is **perfectly vertical (perfectly inelastic)** in this case (not shown below…). A change in taxes does not affect this perfectly inelastic labor supply function.



Problem 2e continued (more work space)

f. **(6 points)** How do the conclusions in part e compare with those in part c? Are they broadly similar? Are they very different? Is it impossible to compare them? **In no more than 60 words,** describe as much as you can about the **economics (do *__not__* simply restate the mathematics you computed above)** when comparing the pair of diagrams.

Solution: Broadly, the difference between part c and part e is that part c is a "microeconomic" analysis, while part e is a "macroeconomic" analysis. More precisely, part c is, intuitively, a purely "slope" argument, rather than both a "slope" and a "level" argument in part e. The analysis in part c is tantamount to analyzing the effects of policy on **just** the labor market (why? – because the analysis there treats consumption as a constant). The analysis in part e instead is tantamount to analyzing **jointly** the effects of policy on labor markets **and** goods markets. To the extent that there are feedback effects between the two markets, there is no reason to think the answers from the analyses must be the same.

The latter is the basis for thinking of the analysis in part c as a "microeconomic" analysis and the analysis in part e as a "macroeconomic" analysis. What this implies is that one way (perhaps the most important way) to understand the difference between "microeconomic" analysis and "macroeconomic" analysis is that the latter routinely considers feedback effects across markets, whereas the former usually does not.

The stark perfectly elastic/perfectly inelastic case first arose in the work of Hansen (1985 *Journal of Monetary Economics*) and Rogerson (1988 *Journal of Monetary Economics*), and has been a staple example, in the sense of being able to easily convey ideas, in macroeconomic analysis since then.

Problem 3: "Hyperbolic" Impatience (i.e., "Hyperbolic" Discounting) and Stock Prices (30 points). In this problem you will study a slight extension of the infinite-period economy from Chapter 8. Specifically, suppose the representative consumer has a lifetime utility function given by

$$u(c_t) + \gamma\beta u(c_{t+1}) + \gamma\beta^2 u(c_{t+2}) + \gamma\beta^3 u(c_{t+3}) + \ldots,$$

in which, as usual, $u(.)$ is the consumer's utility function in any period and β is a number between zero and one that measures the "normal" degree of consumer impatience. **The number γ (the Greek letter "gamma," which is the new feature of the analysis here) is also a number between zero and one, and it measures an "additional" degree of consumer impatience, but one that ONLY applies between period t and period t+1.[1] This latter aspect is reflected in the fact that the factor γ is NOT successively raised to higher and higher powers as the summation grows.**

The rest of the framework is **exactly** as studied in Chapter 8: a_{t-1} is the representative consumer's holdings of stock at the beginning of period t, the nominal price of each unit of stock during period t is S_t, and the nominal dividend payment (per unit of stock) during period t is D_t. Finally, the representative consumer's consumption during period t is c_t and the nominal price of consumption during period t is P_t. As usual, analogous notation describes all these variables in periods t+1, t+2, etc.

The Lagrangian for the representative consumer's utility-maximization problem **(starting, as always, from the perspective of the beginning of period *t*)** is

$$\begin{aligned}
&u(c_t) + \gamma\beta u(c_{t+1}) + \gamma\beta^2 u(c_{t+2}) + \gamma\beta^3 u(c_{t+3}) + \ldots \\
&+\lambda_t\left[Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t\right] \\
&+\gamma\beta\lambda_{t+1}\left[Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1}c_{t+1} - S_{t+1}a_{t+1}\right] \\
&+\gamma\beta^2\lambda_{t+2}\left[Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2}c_{t+2} - S_{t+2}a_{t+2}\right] \\
&+\gamma\beta^3\lambda_{t+3}\left[Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3}c_{t+3} - S_{t+3}a_{t+3}\right] \\
&+\ldots
\end{aligned}$$

NOTE CAREFULLY WHERE THE "ADDITIONAL" IMPATIENCE FACTOR γ APPEARS IN THE LAGRANGIAN.

(OVER)

[1] The idea here, which goes under the name "hyperbolic impatience," is that in the "very short run" (i.e., between period t and period t+1), individuals' degree of impatience may be different from their degree of impatience in the "slightly longer short run" (i.e., between period t+1 and period t+2, say). "Hyperbolic impatience" is a phenomenon that routinely recurs in laboratory experiments in experimental economics and psychology, and has many far-reaching economic, financial, policy, and societal implications.

Problem 3 continued

a. **(4 points)** Compute the first-order conditions of the Lagrangian above with respect to **both** a_t **and** a_{t+1}. (**Note:** There is no need to compute first-order conditions with respect to any other variables.)

Solution: The two FOCs are

$$-\lambda_t S_t + \gamma\beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$$
$$-\gamma\beta\lambda_{t+1}S_{t+1} + \gamma\beta^2\lambda_{t+2}(S_{t+2} + D_{t+2}) = 0$$

b. **(4 points)** Using the first-order conditions you computed in part a, construct **TWO** distinct stock-pricing equations, one for the price of stock in period t, and one for the price of stock in period t+1. Your final expressions should be of the form $S_t = ...$ and $S_{t+1} = ...$ (**Note:** It's fine if your expressions here contain Lagrange multipliers in them.)

Solution: Simply rearranging the two FOCs above and canceling the γ term (along with one β term) in the second FOC, we have

$$S_t = \frac{\gamma\beta\lambda_{t+1}}{\lambda_t}(S_{t+1} + D_{t+1})$$
$$S_{t+1} = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}}(S_{t+2} + D_{t+2})$$

For the questions next, observe that the S_t expression and the S_{t+1} expression are subtly, but importantly, different here. They would be identical to each other (other than the fact that the time subscripts are different, but that is as usual) if and only if $\gamma = 1$. If $\gamma < 1$, which is the case of "hyperbolic impatience," then stock prices are determined in a somewhat "different way" in the "very short run" compared to the "longer short run" or "medium run."

Problem 3 continued

For the remainder of this problem, suppose that it is known that $D_{t+1} = D_{t+2}$, and that $S_{t+1}=S_{t+2}$, and that $\lambda_t = \lambda_{t+1} = \lambda_{t+2}$. **BUT you know nothing else about any other numerical values.**

c. **(5 points).** **IN 50 WORDS OR LESS:** does the above information necessarily imply that the economy is in a steady-state? **Briefly, but carefully, explain why or why not; your response should make clear what the definition of a "steady state" is.** **(Note:** To address this question, it's possible, though not necessary, that you may need to compute other first-order conditions besides the ones you have already computed above.)

Solution: No, none of these statements necessarily imply that the economy is in a steady state, which, recall, means that all real variables become constant and **never** again change. There are two ways of observing that the above information does not imply the economy is in steady state. First, the above statements are all about **nominal** variables, and in a steady state it can be the case that nominal variables continue fluctuating over time, even though all real variables do not. Another way of arriving at the correct conclusion here is that the statements above only refer to periods t, t+1, and t+2. In a steady-state, (real) variables settle down to constant values **forever,** not just for a few time periods.

Problem 3 continued

d. **(5 points) IN 50 WORDS OR LESS:** based on the above information and your stock-price expressions from part b, can you conclude that the period-t stock price (S_t) is higher than S_{t+1}, lower than S_{t+1}, equal to S_{t+1}, or is it impossible to determine? **Briefly and carefully explain the economics (i.e., the economic reasoning, not simply restating the mathematics) of your finding.**

Solution: A bit of an expanded discussion is useful here, since very few people solved this question correctly.

You are given that nominal stock prices, nominal dividends, and the Lagrange multiplier in period t+1 and t+2 are equal to each other. Let's call these common values $\overline{S}$, $\overline{D}$, and $\overline{\lambda}$ (that is, $\overline{S} = S_{t+1} = S_{t+2}$ $\overline{D} = D_{t+1} = D_{t+2}$, and $\overline{\lambda} = \lambda_t = \lambda_{t+1} = \lambda_{t+2}$). Inserting these common values in the period-t+1 stock price equation, we have $\overline{S} = \frac{\beta\overline{\lambda}}{\overline{\lambda}}(\overline{S} + \overline{D})$. Canceling terms, we have that the nominal stock price in period t+1 (and t+2) is $\overline{S} = \beta\overline{S} + \beta\overline{D}$ (which we could of course solve for the stock price as $\overline{S} = \frac{\beta}{1-\beta}\overline{D}$ if we needed to).

Now, using the common values of *S, D*, and the multiplier in the period-t stock price equation gives us $S_t = \gamma\beta(S_{t+1} + D_{t+1}) = \gamma\beta(\overline{S} + \overline{D}) = \gamma(\beta\overline{S} + \beta\overline{D})$. Note that the final term in parentheses is nothing more than $\overline{S}$, hence we have

$$S_t = \gamma\overline{S}.$$

If $\gamma < 1$, then clearly the stock-price in period t is smaller than it is in period t+1 (and period t+2). The economics of this is due to the "hyperbolic impatience" which makes consumers more impatient to purchase consumption in the "very short run" (period t) compared to the "longer short run." All else equal, this means that in the very short run, consumers do not care to save as much (due to their extreme impatience in the very short run), which means their demand for saving – i.e., their demand for stock – is lower. Lower demand for stock means a lower price of stock, all else equal.

Problem 3 continued

Now also suppose that the utility function in every period is $u(c) = \ln c$, and also that the **real interest rate is zero in every period.**

e. **(6 points) IN 60 WORDS OR LESS:** based on the utility function given, the fact that $r = 0$, and the basic setup of the problem described above, **construct TWO marginal rates of substitution (MRS):** the MRS between period-t consumption and period-t+1 consumption, **and** the MRS between period-t+1 consumption and period-t+2 consumption. **(Note:** your analysis is starting from the very beginning of **period t.)**

Solution: This only requires examining the lifetime utility function (the first line of the Lagrangian above). By definition, the MRS between period t consumption and t+1 consumption is $\frac{u'(c_t)}{\gamma\beta u'(c_{t+1})} = \frac{c_{t+1}}{\gamma\beta c_t}$, and the MRS between period t+1 consumption and t+2 consumption is $\frac{\gamma\beta u'(c_{t+1})}{\gamma\beta^2 u'(c_{t+2})} = \frac{u'(c_{t+1})}{\beta u'(c_{t+2})} = \frac{c_{t+2}}{\beta c_{t+1}}$. Note that the form of the two MRS functions is different: the hyperbolic impatience affects the former MRS, but not the latter MRS.

Problem 3 continued

f. **(6 points – Harder) IN 60 WORDS OR LESS:** from the perspective of the very beginning of period t, and based on the two MRS functions you computed in part e and on the fact that *r* = 0 in every period, determine which of the following two consumption growth rates

$$\frac{c_{t+1}}{c_t} \quad \text{OR} \quad \frac{c_{t+2}}{c_{t+1}}$$

is larger. That is, is the consumption growth rate between period t and period t+1 (the fraction on the left) expected to be larger than, smaller than, or equal to the consumption growth rate between period t+1 and period t+2 (the fraction on the right), or is it impossible to determine? **Carefully explain your logic, and briefly explain the economics (i.e., the economic reasoning, not simply restating the mathematics) of your finding.**

Solution: Once again, something a bit expanded compared to the required solution is helpful in explaining the economics.

The basic consumption-savings optimality condition states that the MRS between two consecutive time periods is equated to (1+r). You are told here that r = 0 always. Based on the two MRS functions constructed above, then, it follows immediately that the consumption growth rate between period t and t+1 is **smaller than** the consumption growth rate between period t+1 and period t+2. This follows because $\gamma < 1$. The economics is similar to above: hyperbolic impatience makes consumers consume "much more" in the very short run (i.e., period t), which means that the growth rate of consumption between period t (already a very high consumption period) and t+1 will be low, compared to the similar comparison one period later.

Problem 4: Government in the Two-Period Economy (15 points). Consider an economy that lasts for two periods. Neither the representative consumer nor the government start their lives with any assets (that is, both $a_0 = 0$ and $b_0 = 0$). All taxes that the government levies are lump-sum. In each period, the government has positive government spending (i.e., both $g_1 > 0$ and $g_2 > 0$). Suppose that the real interest rate between period one and period two is zero (i.e., $r = 0$). Finally, suppose that the government lives for the entire two periods.

a. **(5 points) IN 30 WORDS OR LESS: briefly** define/describe what a lump-sum tax is.

Solution: A lump-sum tax is one whose total incidence (i.e., total amount paid) depends in no way at all on any choices/decisions that an individual (consumption, income, or, going outside the model, capital holdings) makes.

b. **(5 points) IN 30 WORDS OR LESS**: suppose that the government is currently planning to collect $t_1 = 3$ and $t_2 = 5$ in taxes in period one and period two, respectively. A policy change is proposed, however, that would reduce period-one taxes to $t_1 = 2$ without changing either g_1 or g_2. If this policy change is enacted, is it possible to numerically compute the amount of tax collections that the government will require in period two? If so, compute it; if not, explain why not.

Solution: Because $r = 0$, the lifetime government budget constraint boils down to simply $g_1 + g_2 = t_1 + t_2$. Thus, before the policy proposal, the government is planning to (needs to) collect $t_1 + t_2 = 3 + 5 = 8$ total units (note these are real goods, because everything here is specified in real terms) in taxes. Because government spending is not changing and $r = 0$, reducing period-1 tax collections by one unit necessarily means period-2 tax collection must rise by one unit – hence $t_2 = 6$ if the policy change is enacted.

Problem 4 continued

c. **(5 points) IN 30 WORDS OR LESS**: if the proposed policy change described in part b is enacted, how will it affect consumers' period-one optimal choices of consumption? Specifically, will it increase period-one consumption, decrease it, leave it unchanged, or is it impossible to tell? Briefly discuss/explain.

Solution: The tax change will have no effect at all because Ricardian Equivalence applies – the representative consumer will simply save the entire period-1 tax cut in anticipation of the tax hike which is coming in period 2.

END OF EXAM