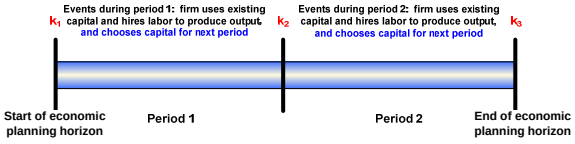
FIRMS IN THE TWO-PERIOD FRAMEWORK (CONTINUED)

OCTOBER 26, 2011

BASICS

- **Timeline of events**



- **Notation**
  - k_1: capital used for production in period 1 (decided upon in "period 0")
  - n_1: labor used for production in period 1
  - w_1: real wage rate for labor in period 1 ($w_1 = W_1/P_1$)
  - i: nominal interest rate
  - P_1: nominal price of output produced and sold by firm in period 1
    AND nominal price of one unit of capital bought by the firm in period 1 for use in period 2 (recall time to build…)

    Underlying assumption/view of world: ***capital goods*** are not necessarily "distinct" from ***consumption goods*** (i.e., computers purchased by both firms and individual consumers)

FIRM PROFIT MAXIMIZATION

- A ***dynamic*** profit maximization problem
  - Because firm exists for both periods
  - All analysis conducted from the perspective of the very beginning of period 1
  - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits

- Dynamic profit function
  - (specified in nominal terms – could specify in real terms…)

Period-1 profits | (PDV of) period-2 profits | = 0

$$P_1 f(k_1, n_1) + P_1 k_1 - P_1 w_1 n_1 - P_1 k_2 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i}$$

Total revenue in period 1 (price x output)	Value of pre-existing capital (an asset for firms)	Total labor cost in period 1	Total cost of buying capital for period 2 (time to build → must purchase period-2 capital in period 1)	Total revenue in period 2 (price x output)	Value of pre-existing capital (an asset for firms)	Total labor cost in period 2	Total cost of buying capital for period 3 (time to build → must purchase period-3 capital in period 2)

- Two-period model: $k_3 = 0$ (no machines needed in "period 3")

FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1, n_1) + P_1 k_1 - P_1 w_1 n_1 - P_1 k_2 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i}$$ (= 0)

- FOCs with respect to n_1, n_2, k_2

Identical except for time subscripts

with respect to n_1: $P_1 f_n(k_1, n_1) - P_1 w_1 = 0$ — Equation 1

with respect to n_2: $\frac{P_2 f_n(k_2, n_2)}{1+i} - \frac{P_2 w_2}{1+i} = 0$ — Equation 2

with respect to k_2:

FIRM PROFIT MAXIMIZATION

- **Re-express equation 3**

$$-P_1 + \frac{P_2 f_k(k_2,n_2)}{1+i} + \frac{P_2}{1+i} = 0$$

Divide by P_1

$$\frac{P_2 f_k(k_2,n_2)}{P_1(1+i)} + \frac{P_2}{P_1(1+i)} = 1$$

Group terms informatively

$$\left(\frac{P_2}{P_1}\right)\left(\frac{1}{1+i}\right) f_k(k_2,n_2) + \left(\frac{P_2}{P_1}\right)\left(\frac{1}{1+i}\right) = 1$$

$P_2/P_1 = 1 + \pi_2$

$$\left(\frac{1+\pi_2}{1+i}\right) f_k(k_2,n_2) + \left(\frac{1+\pi_2}{1+i}\right) = 1$$

Fisher equation

$$\frac{f_k(k_2,n_2)}{1+r} + \frac{1}{1+r} = 1$$

Multiply by $1+r$

$$f_k(k_2,n_2) + 1 = 1 + r$$

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Multiply by $1+r$

$$f_k(k_2,n_2) + 1 = 1 + r$$

$$\boxed{f_k(k_2,n_2) = r}$$

Equivalent / alternative representation of firm profit-maximizing condition for capital

FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1, n_1) + P_1 k_1 - P_1 w_1 n_1 - P_1 k_2 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i}$$ (= 0)

- **FOCs with respect to n_1, n_2, k_2**

Identical except for time subscripts

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- with respect to k_2: $-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} = 0$ — equivalent ↔ $f_k(k_2, n_2) = r$ — Equation 3

FIRM PROFIT MAXIMIZATION

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- **Profit-maximizing labor hiring implies**

$$f_n(k_1, n_1) = w_1 \quad \text{AND} \quad f_n(k_2, n_2) = w_2$$

- **Profit-maximizing capital purchases (for the future…) implies**

$$f_k(k_2, n_2) = r$$

FIRM PROFIT MAXIMIZATION

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- **Marginal product of labor**
  - $f_n(k_t, n_t)$
  - Sometimes denote by mpn_t
- **Marginal product of capital**
  - $f_k(k_t, n_t)$
  - Sometimes denote by mpk_t

These FOCs are foundation for:
1. Labor Demand
2. Capital / Investment Demand

COBB-DOUGLAS PRODUCTION FUNCTION

- **A commonly-used functional form in modern quantitative macroeconomic analysis**

$$f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}$$ (saw Cobb-Douglas utility function on Practice Problem Set 1)

- **Describes the empirical relationship between aggregate GDP, aggregate capital, and aggregate labor quite well**
- $\alpha \in (0,1)$ **measures capital's share of output**
  - **Hence** $(1-\alpha) \in (0,1)$ **measures labor's share of output**
  - **Interpretation**
    - **The relative importance of (either) capital (or labor) in the production process**
  - **Estimates for U.S. economy:** $\alpha \approx 0.3$
  - **Estimates for Chinese economy:** $\alpha \approx 0.15$ **(not (yet) a very capital-rich economy)**
- **Cobb-Douglas form useful for illustrating factor demands**
  - $mpn_t = f_n(k_t, n_t) = (1-\alpha) k_t^{\alpha} n_t^{-\alpha}$
  - $mpk_t = f_k(k_t, n_t) = \alpha k_t^{\alpha-1} n_t^{1-\alpha}$

MICRO-LEVEL LABOR DEMAND

- **Firm-level demand for labor defined by the relation**

Follows from Equation 1 and Equation 2 $w_t = (1-\alpha)k_t^{\alpha} n_t^{-\alpha} (= mpn_t)$ for both $t = 1$ and $t = 2$

↓ Because exponent ($-\alpha$) is a negative number, can move to denominator

$$w_t = (1-\alpha)\left(\frac{k_t}{n_t}\right)^{\alpha}$$

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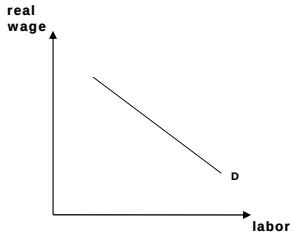
A RELATIONSHIP BETWEEN w_t and n_t



LABOR DEMAND

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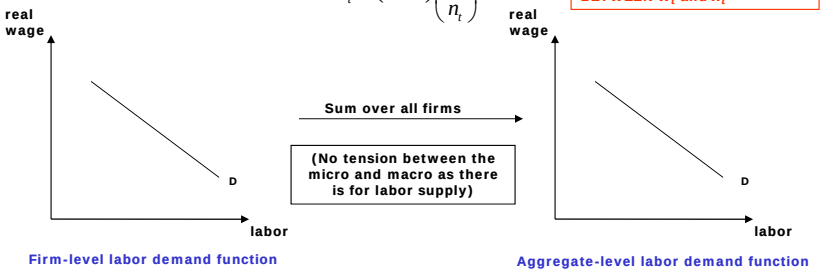

Firm-level labor demand function

Aggregate-level labor demand function

- **Completes picture of the aggregate labor market**

MICRO-LEVEL CAPITAL DEMAND

- **Firm-level demand for capital defined by the relation**

Follows from Equation 3 (will see it soon…)

$$r_t = \alpha k_t^{\alpha-1} n_t^{1-\alpha} (= mpk_t)$$

Because exponent ($\alpha - 1$) is a negative number, can move to denominator

$$r_t = \alpha\left(\frac{n_t}{k_t}\right)^{1-\alpha}$$

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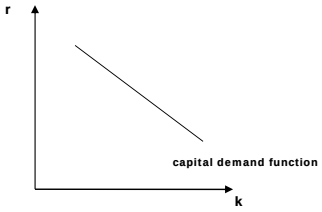
A RELATIONSHIP BETWEEN r_t and k_t



CAPITAL DEMAND

- **Firm-level demand for capital defined by the relation**

Follows from Equation 3 (will see it soon…)

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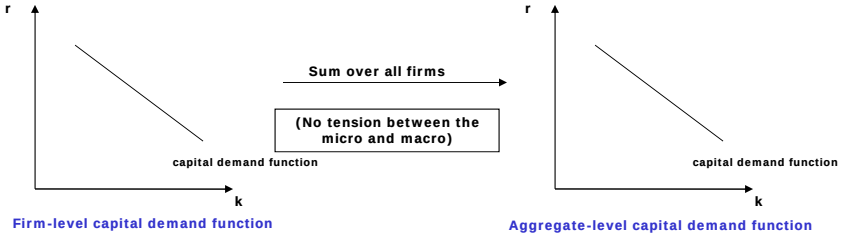
A RELATIONSHIP BETWEEN r_t and k_t



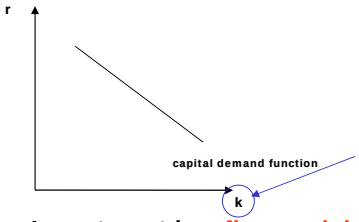
Firm-level capital demand function

Aggregate-level capital demand function

- **(Almost…) completes picture of the aggregate capital market**

FROM CAPITAL DEMAND TO INVESTMENT DEMAND

- **Capital is a stock variable**



Want investment (a flow) to show up here, not capital (a stock)

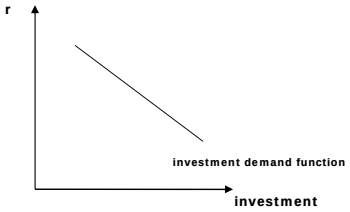
Investment is ***change*** in capital stock between consecutive periods

$inv_1 = k_2 - k_1$

At start of period 1, k_1 cannot be changed. Thus any rise in demand for k_2 is reflected one-for-one in a rise in inv_1.

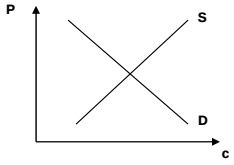
→ Capital demand and investment demand functions have same shape

- **Investment is a flow variable**

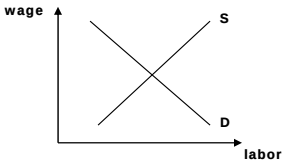


THE THREE MACRO (AGGREGATE) MARKETS

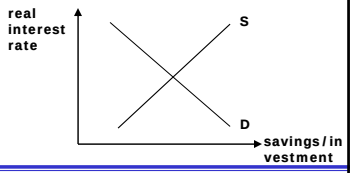
- **Goods Markets**
  - Demand derived from C-L framework
    (For S, have to consider how aggregate NOMINAL P is determined…Chapter 14)



- **Labor Markets**
  - Supply derived from C-L framework
  - Demand derived from firm theory in C-S framework



- **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**
  - Supply derived from C-S framework
    Demand derived from firm theory in C-S framework



REAL INTEREST RATE

- ***r*** **a key variable for macroeconomic analysis**
- **Chapter 4: *r* measures the price of period-1 consumption in terms of period-2 consumption**
- **Chapter 8: *r* reflects degree of impatience (in the long run)**
- ***r*** **often reflects rate of consumption growth between periods**
- **Now: *r* measures the price of capital (machine and equipment) purchases by firms**
  - **Reflects (real!) opportunity cost of sinking funds into capital *today* that won't bear fruit (i.e., help produce output) until the *future***
    - **Regardless of whether firm actually has to "borrow" to purchase capital**

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    - **Regardless of whether firm actually has to "borrow" to purchase capital**
  - **Can see it mathematically**

$$-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} = 0$$

Equation 3 (FOC on k_2)

$$f_k(k_2, n_2) = r$$

When firms make optimal investment decisions → *r = mpk*