THE FINANCIAL ACCELERATOR: FINANCIAL MARKETS AND THE MACROECONOMY

NOVEMBER 28, 2011

FINANCIAL ACCELERATOR

- "Financial accelerator" framework
  - The most widely-used and applied framework in macroeconomic theory and policy for thinking about financial markets
  - Developed in series of studies by Bernanke and Gertler in late 1980's and early 1990's
- Popular-press language
  - "Financial accelerator"
  - "Financial feedback loops"
  - "Loan spirals"
- Describes well many of the financial-macroeconomic linkages underpinning the dynamics of
  - The Great Depression
  - Current macroeconomic conditions
- Will develop idea in context of firm theory (Chapter 6)
- Can also develop idea in context of consumer theory (Chapter 3, Chapter 4, Chapter 8)
  - Recall "credit constraint" analysis of consumption/savings decisions (Chapter 3 and 4)

OUTLINE OF FRAMEWORK

Major ideas underlying Financial Accelerator Framework

1. **Firms' financial assets (i.e., stocks and bonds) matter for their ability to purchase physical assets (i.e., machines and equipment)**

2. **Market prices of financial assets matter for firm financing constraints**

3. **Government regulation affects the linkage between financial markets and real (i.e., goods and physical capital) markets *through* financing constraints**

OUTLINE OF FRAMEWORK

Four Building Blocks of the Financial Accelerator Framework

1. **Two-Period Model of Firm Profit Maximization**
   - **Based on Chapter 6**
   - **Enriched to allow for both physical assets (machines and equipment) and financial assets (stocks and bonds)**

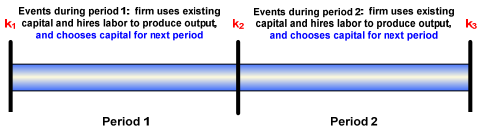
2. **Financing Constraint – conceptually, the key building block**
   - **Quantity of physical capital firms can purchase depends on the market value (i.e., price x quantity) of their financial assets**
   - **Reflects market and regulatory structures designed to mitigate informational asymmetries**
   - **(Recall basic Chapter 6 theory of firms featured no constraints on firm profit maximization)**

3. **Government Regulation/Oversight of Financial Relationships**

4. **Relationship between Firm Profits and Dividends**

BASIC FIRM THEORY

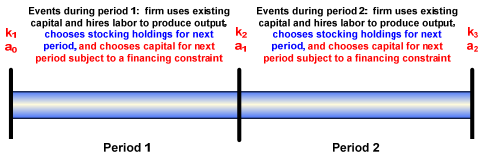
- **Timeline of events**



- **Notation**
  - k_1: capital used for production in period 1 (decided upon in "period 0")
  - n_1: labor used for production in period 1
  - w_1: real wage rate for labor in period 1 ($w_1 = W_1/P_1$)
  - i: nominal interest rate (between period 1 and period 2)
  - P_1: nominal price of output produced and sold by firm in period 1
    AND nominal price of one unit of capital bought by the firm in period 1 for use in period 2

ENRICHING THE BASIC FIRM THEORY

- **Timeline of events**



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  - a_0: real wealth (stock) holdings at beginning of period 1/end of period 0

The "defining features" of stock

  - S_1: nominal price of a unit of stock in period 1
  - D_1: nominal dividend paid in period 1 by each unit of stock held at the start of period 1

ENRICHING THE BASIC FIRM THEORY

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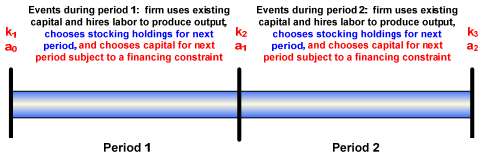

- **Notation**
  - k_2: capital used for production in period 2 (decided upon in period 1)
  - n_2: labor used for production in period 2
  - w_2: real wage rate for labor in period 2 ($w_2 = W_2/P_2$)
  - i: nominal interest rate (between period 1 and period 2)
  - P_2: nominal price of output produced and sold by firm in period 2
    AND nominal price of one unit of capital bought by the firm in period 2 for use in period 3
  - a_1: real wealth (stock) holdings at beginning of period 2/end of period 1

The "defining features" of stock

  - S_2: nominal price of a unit of stock in period 2
  - D_2: nominal dividend paid in period 2 by each unit of stock held at the start of period 2
  - π_2: net inflation rate between period 1 and period 2 (recall: $\pi_2 = P_2/P_1 - 1$)

RATES OF RETURN

- "Interest rates" can be defined for any type of asset
  - There is no ***single*** interest rate in the economy

- Interpret/understand the ***two*** types of "interest rates" that co-exist in this richer theory of firm profit maximization
  - i: nominal interest rate on bonds
    - Recall from Chapter 14

$$1+i=\frac{1}{P_1^b}$$

    - Thus can think of bonds (one ***type*** of financial asset) as being in the background of the analysis

  - i^{STOCK}: nominal return on stock – i.e., "interest rate on stock" (though bad terminology)
    - Define according to

$$1+i^{STOCK}=\frac{S_2+D_2}{S_1}$$

    - Measures the net dollar return (in period 2) on one share of stock (whose purchase price was S_1 in period 1)

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    REAL INTEREST RATE ON GOVERNMENT BONDS: A "RISKLESS" ASSET
    - Recall from Chapter 14

      $1+i=\frac{1}{P_1^b}$ ↔ (can rewrite as) $i=\frac{1}{P_1^b}-1$ ↔ (express as *real* interest rate) $1+r=\frac{1+i}{1+\pi}$
    - Thus can think of bonds (one ***type*** of financial asset) as being in the background of the analysis
  - i^{STOCK}: nominal return on stock – i.e., "interest rate on stock" (though bad terminology)

    REAL INTEREST RATE ON STOCKS: A "RISKY" ASSET
    - Define according to

      $1+i^{STOCK}=\frac{S_2+D_2}{S_1}$ ↔ (can rewrite as) $i^{STOCK}=\frac{S_2+D_2}{S_1}-1$ ↔ (express as *real* interest rate) $1+r^{STOCK}=\frac{1+i^{STOCK}}{1+\pi}$
    - Measures the net dollar return (in period 2) on one share of stock (whose purchase price was S_1 in period 1)
- Can distinguish two measures of real interest rates in this framework

FIRM PROFIT FUNCTION

- A ***dynamic*** profit maximization problem
  - Because firm exists for both periods
  - All analysis conducted from the perspective of the very beginning of period 1
  - → Must consider present-discounted-value (PDV) of lifetime (i.e., two-period) profits
- Dynamic profit function
  - (specified in nominal terms – could specify in real terms…)

Period-1 profits

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1$$

- $P_1 f(k_1, n_1)$: Total revenue in period 1 (price x output)
- $P_1 k_1$: Value of pre-existing physical capital (an asset for firms)
- $(S_1 + D_1) a_0$: Value (inclusive of dividends) of pre-existing financial assets (i.e., stock-holdings in *other* firms)
- $P_1 w_1 n_1$: Total labor cost in period 1
- $P_1 k_2$: Total cost of buying physical capital for period 2 (time to build → must purchase period-2 capital in period 1)
- $S_1 a_1$: Total cost of buying financial assets (i.e., stock-holdings in *other* firms) for period 2

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As usual: no physical or financial assets needed for "period 3"

= 0 = 0

Period-1 profits

(PDV of) period-2 profits

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

Total revenue in period 1 (price x output)

Value of pre-existing physical capital (an asset for firms)

Total labor cost in period 1

Total cost of buying physical capital for period 2 (time to build → must purchase period-2 capital in period 1)

Total revenue in period 2 (price x output)

Value of pre-existing physical capital (an asset for firms)

Total labor cost in period 2

Total cost of buying physical capital for period 3 (time to build → must purchase period-3 capital in period 2)

Value (inclusive of dividends) of pre-existing financial assets (i.e., stock-holdings in *other* firms)

Total cost of buying financial assets (i.e., stock-holdings in *other* firms) for period 2

Value (inclusive of dividends) of pre-existing financial assets (i.e., stock-holdings in *other* firms)

Total cost of buying financial assets (i.e., stock-holdings in *other* firms) for period 3

INFORMATIONAL ASYMMETRIES

- "Informational asymmetries" pervasive in borrowing/lending relationships

- Borrower (whether consumer, firm, or financial institution) ***much*** more likely to know his own ability/willingness to repay a loan
  - Lenders only know little about the "quality" or "trustworthiness" of a borrower
  - Asymmetry of information – cannot be eliminated

- To mitigate consequences of informational asymmetries, lenders often require borrower to have a stake in "succeeding" in the project/purpose for which funds are being borrowed
  - Consumers
    - e.g., down payment on house purchase
    - e.g., down payment on car purchase

    - If stop making payments on house or car
      - Borrower loses down payment (in addition to the car or house…)…
      - Affects individual's incentives ***before*** borrowing

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  - Consumers
    - e.g., down payment on house purchase
    - e.g., down payment on car purchase
    - Total amount of loan (typically) depends on individual's collateral
  - Firms
    - Capital investment (factories, technology upgrades, etc) outlays
    - Payroll outlays
    - Financing inventories
    - Total amount of loan (often) depends on firm's collateral

"Working capital"

  - Financial institutions: borrow in order to make (big) loans
    - By raising "small" quantities of funds from many different sources

FINANCING CONSTRAINT

- Capture this idea through a financing constraint on firm's ability to purchase capital between period 1 and period 2
- Financing constraint
  - Total expenditures on period-1 physical investment must be equal to market value of firm's financial (stock) holdings
  - (Technically, smaller than or equal to, so an inequality constraint…but will only analyze constraint with equality)

$$P_1 \cdot inv_1 = S_1 \cdot a_1$$

$inv_1 = k_2 - k_1$ (investment is ***change in*** quantity of physical capital)

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

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  - Important: a_1 appears in the financing constraint, *not* a_0
    - Idea this assumption captures: firm might try to strategically manipulate the value of *financial* assets it holds in order to affect the quantity of *physical* investment it can engage in
    - (From the perspective of beginning of period 1, a_1 has not yet been chosen, whereas a_0 is pre-determined)

GOVERNMENT OVERSIGHT OF FINANCIAL MARKETS

- Government oversight of informational asymmetries in borrower/lender relationships
  - Filing of proper documentation
  - Full disclosure ("truth-in-lending") laws
  - Direct lending in some markets
  - …
- Capture government Regulation of financial dealings in our framework in very simple way
  - Firm can only borrow up to a multiple *R* of the market value of its financial assets for physical investment purposes
  - e.g., if government regulates that expenditures on investment cannot be larger than 5 times market value of financial assets, *R* = 5
- Terminology: *R* is leverage ratio
  - Will think of it as government regulation…
  - …but can and does also reflect market and institutional arrangements

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Government regulation *R*

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

Impose this financing constraint on firm profit maximization problem

FINANCIAL ACCELERATOR FRAMEWORK

- Four Building Blocks of the Financial Accelerator Framework

1. Firm Profit Function

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

$= 0$ $= 0$

2. Financing Constraint

$$P_1 \cdot (k_2 - k_1) = S_1 \cdot a_1$$

3. Government Regulation of Financial Relationships (imposition of *R* on financing constraint)

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

4. Relationship between firm profits and dividends

LATER

FIRM PROFIT MAXIMIZATION

Maximize two-period profits

= 0 = 0

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

Subject to financing constraint

$$P_1 \cdot (k_2 - k_1) = R \cdot S_1 \cdot a_1$$

Construct Lagrangian

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i}$$
$$+ \lambda \left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1) \right]$$

Lagrange multiplier on financing constraint

FIRM PROFIT MAXIMIZATION

Maximize two-period profits

= 0 = 0

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$$+ \lambda \left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1) \right]$$

Lagrange multiplier on financing constraint

CRUCIAL OBSERVATION: in basic firm theory (i.e., Chapter 6), value of this multiplier was....

λ = 0 i.e., there was no financing constraint!

NEXT TIME: will think about what regulatory and/or market features make the financing constraint effectively "disappear" (i.e., cause λ = 0)

FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} + \lambda\left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)\right]$$

- **FOCs with respect to n_1, n_2**

Identical except for time subscripts

- with respect to n_1: $P_1 f_n(k_1, n_1) - P_1 w_1 = 0$ — Equation 1
- with respect to n_2: $\frac{P_2 f_n(k_2, n_2)}{1+i} - \frac{P_2 w_2}{1+i} = 0$ — Equation 2

  - Financing constraint does not affect profit-maximizing choices of labor hiring…
  - …thus same analysis from Chapter 6 of labor demand curve, etc, applies

- **FOCs with respect to k_2, a_1**

FIRM PROFIT MAXIMIZATION

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 + \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} + \lambda\left[R \cdot S_1 \cdot a_1 - P_1 \cdot (k_2 - k_1)\right]$$

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  - …thus same analysis from Chapter 6 of labor demand curve, etc, applies

- **FOCs with respect to k_2, a_1**
  - The interesting aspects of this framework
  - Next: when do financing constraints matter?
  - Next: the financial accelerator framework in action