REVIEW OF CONSUMER THEORY (CONTINUED)

SEPTEMBER 4, 2014

UTILITY FUNCTIONS

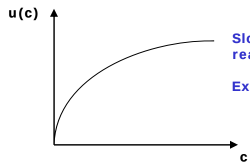
- Describe how much "happiness" or "satisfaction" an individual experiences from "consuming" goods – the benefit of consumption
- Marginal Utility
  - The extra total utility resulting from consumption of a small/incremental extra unit of a good
  - Mathematically, the (partial) slope of utility with respect to that good

Alternative notation: du/dc OR $u'(c)$ OR $u_c(c)$ OR $u_1(c)$

- One-good case: $u(c)$, with $du/dc > 0$ and $d^2u/dc^2 < 0$
  - Recall interpretation: strictly increasing at a strictly decreasing rate
  - Diminishing marginal utility
- Two-good case: $u(c_1, c_2)$, with $u_i(c_1, c_2) > 0$ and $u_{ii}(c_1, c_2) < 0$ for each of $i = 1,2$
  - Utility strictly increasing in each good individually (partial)
  - Diminishing marginal utility in each good individually
- Easily extends to N-good case: $u(c_1, c_2, c_3, c_4,\ldots, c_N)$

UTILITY FUNCTIONS

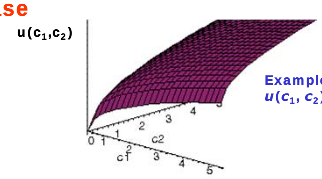
- **One-good case**



Slope (marginal utility) asymptotes to (but never reaches…) zero

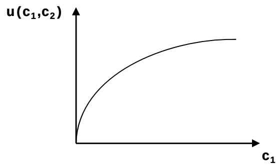
Example: $u(c)$ = ln c or $u(c)$ = sqrt(c)

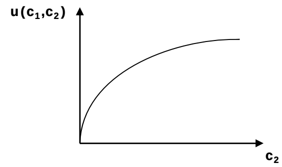
- **Two-good case**



Example: $u(c_1, c_2)$ = ln c_1 + ln c_2 or $u(c_1, c_2)$ = sqrt(c_1) + sqrt(c_2)

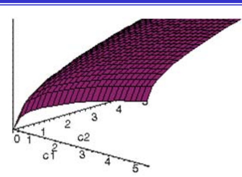
Viewed in good-by-good space



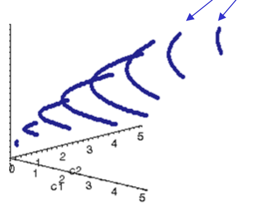


UTILITY FUNCTIONS

Alternative views

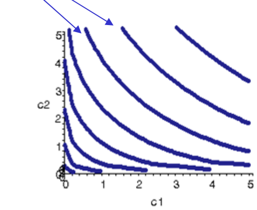


Emphasizing the contours



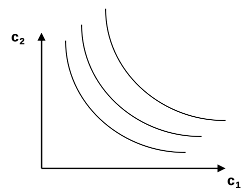
Indifference Curve: the set of all consumption bundles that deliver a particular level of utility / happiness

Viewing only the contours



UTILITY FUNCTIONS

- **Marginal Rate of Substitution (MRS)**
  - **Maximum quantity of one good consumer is willing to give up to obtain one extra unit of the other good**
  - **Graphically, the (negative of the) slope of an indifference curve**
  - **MRS is itself a function of c_1 and c_2 (i.e., $MRS(c_1, c_2)$)**
  - **MRS equals ratio of marginal utilities**
    - $$MRS(c_1, c_2) = \frac{u_1(c_1, c_2)}{u_2(c_1, c_2)}$$
    - **Using Implicit Function Theorem (see Practice Problem Set 1)**



- **Summary: whether graphically- or mathematically-formulated, utility functions describe the benefit side of consumer optimization**

BUDGET CONSTRAINTS

- **Describe the cost side of consumption**
- **One-good case (trivial): $Pc = Y$**
  - **Assume income Y is taken as given by consumer for now…**
- **Two-good case (interesting): $P_1c_1 + P_2c_2 = Y$**
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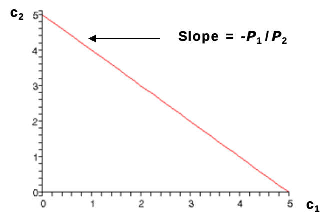
Isolate c_2 to graph the budget constraint

$$P_1c_1 + P_2c_2 = Y$$

$$P_2c_2 = -P_1c_1 + Y$$

$$c_2 = -\frac{P_1}{P_2}c_1 + \frac{Y}{P_2}$$

Plotted in 2D-consumption-space



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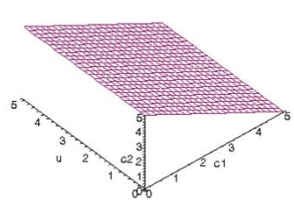
Plotted in 3D-consumption-space



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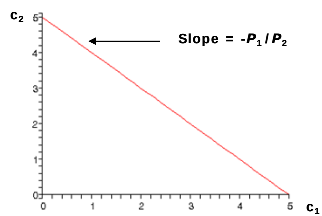
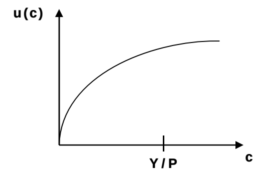


CONSUMER OPTIMIZATION

- **Consumer's decision problem: maximize utility subject to budget constraint – bring together both cost side and benefit side**
- **One-good case**
  - **Optimal *c* = ?...**



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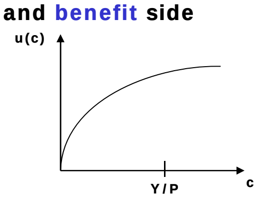
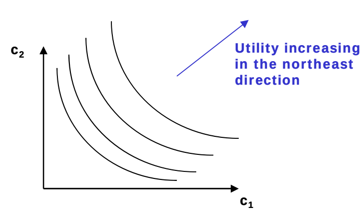


- **Two-good case**
  - **How to optimally allocate *Y* across the two goods c_1 and c_2?**
  - **A non-trivial decision problem…**



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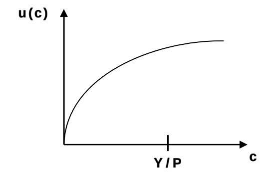

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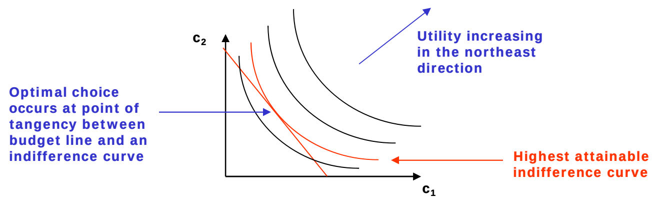

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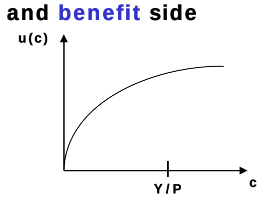

- **Two-good case**
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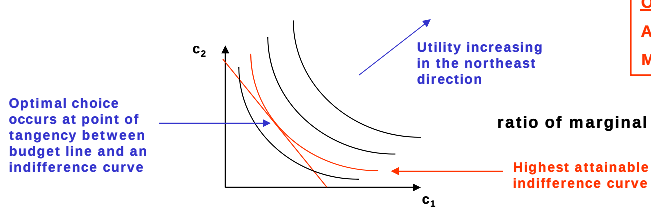
OPTIMALITY CONDITION:
At the optimal choice,
MRS = slope of budget line

ratio of marginal utilities = price ratio



LAGRANGE ANALYSIS

- **Consumer optimization a constrained optimization problem**
  - **Maximize some function (economic application: utility function)…**
  - **…taking into account some restriction on the objects to be maximized over (economic application: budget constraint)**
- **Lagrange Method: mathematical tool to solve constrained optimization problems**

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- **Lagrange Method: mathematical tool to solve constrained optimization problems**

- **General mathematical formulation**
  - **Choose (*x*, *y*) to maximize a given objective function *f*(*x*,*y*)…**
  - **…subject to the constraint *g*(*x*,*y*) = 0 (Note formulation of constraint)**
  - **Step 1: Construct Lagrange function** — Lagrange multiplier (λ)

$$L(x, y, \lambda) = f(x, y) + \lambda g(x, y)$$

Lagrange Analysis

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    Lagrange multiplier

    $$L(x,y,\lambda) = f(x,y) + \lambda g(x,y)$$

  - Step 2: Compute first-order conditions with respect to *x*, *y*, and *λ*

    1) $f_x(x,y) + \lambda g_x(x,y) = 0$

    2) $f_y(x,y) + \lambda g_y(x,y) = 0$

    3) $g(x,y) = 0$

    Rationale: setting first derivatives to zero isolates maxima (or minima…technically, need to check second-order condition…)

The Mathematics of Consumer Theory

Lagrange Analysis

- Step 3: Solve the system of first-order conditions for *x*, *y*, and *λ*
  - Often most interested in simply eliminating the multiplier…
  - From eqn 1), isolate *λ*: $\lambda = -\dfrac{f_x(x,y)}{g_x(x,y)}$
  - Insert expression for *λ* in eqn 2): $f_y(x,y) - \dfrac{f_x(x,y)}{g_x(x,y)} g_y(x,y) = 0$

Lagrange Analysis

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  - **Rearrange**
  - **Optimality condition: at the optimum (x^*, y^*)**

$$\frac{f_x(x^*,y^*)}{f_y(x^*,y^*)} = \frac{g_x(x^*,y^*)}{g_y(x^*,y^*)}$$

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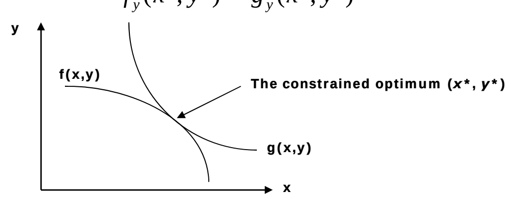
Graphical interpretation: at the constrained optimum, the function $f(.)$ is tangent to the function $g(.)$



LAGRANGE ANALYSIS

- Apply Lagrange tools to consumer optimization
- Objective function: $u(c_1,c_2)$
- Constraint: $g(c_1,c_2) = Y - P_1c_1 - P_2c_2 = 0$

- **Step 1:** Construct Lagrange function

$$L(c_1,c_2,\lambda) = u(c_1,c_2) + \lambda\left[Y - P_1c_1 - P_2c_2\right]$$

- **Step 2:** Compute first-order conditions with respect to c_1, c_2, λ

The Mathematics of Consumer Theory

LAGRANGE ANALYSIS

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- **Step 3:** Solve (focus on eliminating multiplier from eqns 1 & 2)

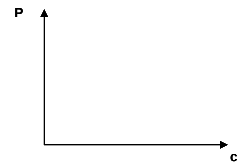
$$\frac{u_1(c_1^*,c_2^*)}{u_2(c_1^*,c_2^*)} = \frac{P_1}{P_2}$$

OPTIMALITY CONDITION

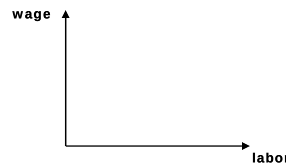
i.e., MRS = price ratio

THE THREE MACRO (AGGREGATE) MARKETS

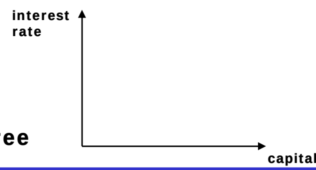
- Goods Markets



- Labor Markets



- Financial / Capital / Savings / Asset Markets



- **Will put micro-foundations under all three**