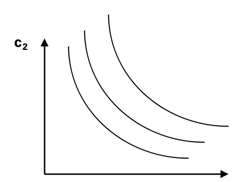
REVIEW OF CONSUMER THEORY (CONTINUED)

SEPTEMBER 7, 2011

UTILITY FUNCTIONS

- **Marginal Rate of Substitution (MRS)**
  - **Maximum quantity of one good consumer is willing to give up to obtain one extra unit of the other good**
  - **Graphically, the (negative of the) slope of an indifference curve**

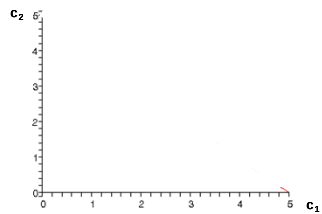


  - **MRS is itself a function of c_1 and c_2 (i.e., $MRS(c_1, c_2)$)**
  - **MRS equals ratio of marginal utilities**
    - $$MRS(c_1,c_2)=\frac{u_1(c_1,c_2)}{u_2(c_1,c_2)}$$
    - **Using Implicit Function Theorem (see Practice Problem Set 1)**
- **Summary: whether graphically- or mathematically-formulated, utility functions describe the benefit side of consumer optimization**

BUDGET CONSTRAINTS

- Describe the **cost** side of consumption
- **One-good case (trivial):** $Pc = Y$
  - Assume income Y is taken as given by consumer for now…
- **Two-good case (interesting):** $P_1c_1 + P_2c_2 = Y$
  - Assume income Y is taken as given by consumer for now…

Plotted in 2D-consumption-space



BUDGET CONSTRAINTS

- Describe the **cost** side of consumption
- **One-good case (trivial):** $Pc = Y$
  - Assume income Y is taken as given by consumer for now…
- **Two-good case (interesting):** $P_1c_1 + P_2c_2 = Y$
  - Assume income Y is taken as given by consumer for now…

Isolate c_2 to graph the budget constraint

$$P_1c_1 + P_2c_2 = Y$$

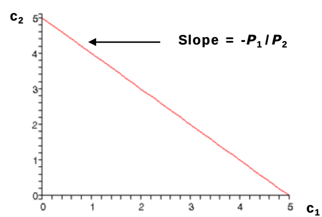
$$\downarrow$$

$$P_2c_2 = -P_1c_1 + Y$$

$$\downarrow$$

$$c_2 = -\frac{P_1}{P_2}c_1 + \frac{Y}{P_2}$$

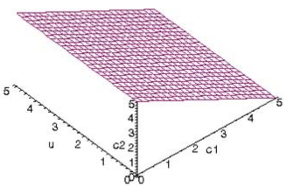
Plotted in 2D-consumption-space



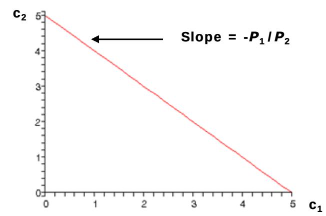
BUDGET CONSTRAINTS

- Describe the **cost** side of consumption
- **One-good case (trivial):** $Pc = Y$
  - Assume income Y is taken as given by consumer for now…
- **Two-good case (interesting):** $P_1c_1 + P_2c_2 = Y$
  - Assume income Y is taken as given by consumer for now…

Plotted in 3D-consumption-space

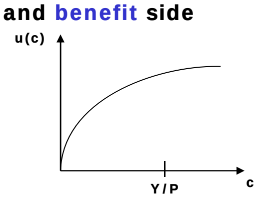


Plotted in 2D-consumption-space



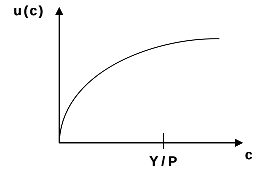
CONSUMER OPTIMIZATION

- **Consumer's decision problem:** maximize utility subject to budget constraint – bring together both **cost** side and **benefit** side
- **One-good case**

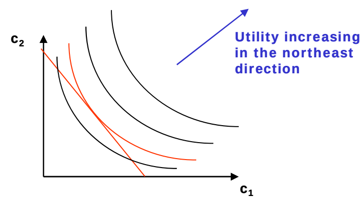


CONSUMER OPTIMIZATION

- **Consumer's decision problem:** maximize utility subject to budget constraint – bring together both cost side and benefit side
- **One-good case**

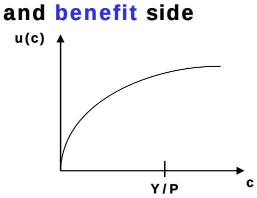


- **Two-good case**
  - How to optimally allocate Y across the two goods c_1 and c_2?
  - A non-trivial decision problem…



CONSUMER OPTIMIZATION

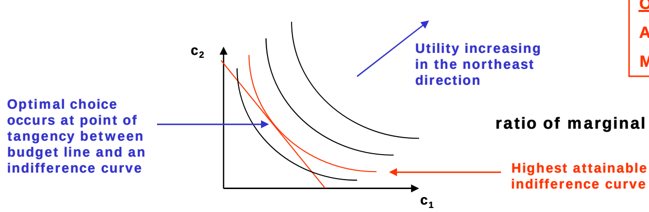
- **Consumer's decision problem:** maximize utility subject to budget constraint – bring together both cost side and benefit side
- **One-good case**



- **Two-good case**
  - How to optimally allocate Y across the two goods c_1 and c_2?
  - A non-trivial decision problem…

OPTIMALITY CONDITION:
At the optimal choice,
MRS = slope of budget line

ratio of marginal utilities = price ratio



4

LAGRANGE ANALYSIS

- Consumer optimization a constrained optimization problem
  - Maximize some function (economic application: utility function)…
  - …taking into account some restriction on the objects to be maximized over (economic application: budget constraint)
- Lagrange Method: mathematical tool to solve constrained optimization problems
- General mathematical formulation
  - Choose (*x*, *y*) to maximize a given objective function *f*(*x*,*y*)…
  - …subject to the constraint *g*(*x*,*y*) = 0 (Note formulation of constraint)

LAGRANGE ANALYSIS

- Consumer optimization a constrained optimization problem
  - Maximize some function (economic application: utility function)…
  - …taking into account some restriction on the objects to be maximized over (economic application: budget constraint)
- Lagrange Method: mathematical tool to solve constrained optimization problems
- General mathematical formulation
  - Choose (*x*, *y*) to maximize a given objective function *f*(*x*,*y*)…
  - …subject to the constraint *g*(*x*,*y*) = 0 (Note formulation of constraint)
  - Step 1: Construct Lagrange function — Lagrange multiplier

$$L(x,y,\lambda) = f(x,y) + \lambda g(x,y)$$

  - Step 2: Compute first-order conditions with respect to *x*, *y*, and *λ*

1) $f_x(x,y) + \lambda g_x(x,y) = 0$
2) $f_y(x,y) + \lambda g_y(x,y) = 0$
3) $g(x,y) = 0$

Rationale: setting first derivatives to zero isolates maxima (or minima…technically, need to check second-order condition…)

LAGRANGE ANALYSIS

- **Step 3:** **Solve the system of first-order conditions for *x*, *y*, and *λ***
  - **Often most interested in simply eliminating the multiplier…**
  - **From eqn 1), isolate *λ*:** $\lambda = -\frac{f_x(x,y)}{g_x(x,y)}$
  - **Insert expression for *λ* in eqn 2):** $f_y(x,y) - \frac{f_x(x,y)}{g_x(x,y)} g_y(x,y) = 0$

LAGRANGE ANALYSIS

- **Step 3:** **Solve the system of first-order conditions for *x*, *y*, and *λ***
  - **Often most interested in simply eliminating the multiplier…**
  - **From eqn 1), isolate *λ*:** $\lambda = -\frac{f_x(x,y)}{g_x(x,y)}$
  - **Insert expression for *λ* in eqn 2):** $f_y(x,y) - \frac{f_x(x,y)}{g_x(x,y)} g_y(x,y) = 0$
  - **Rearrange**
  - **Optimality condition:** **at the optimum (*x**, *y**)**

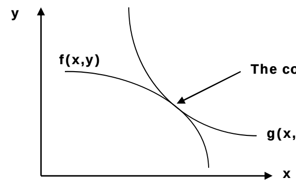
$$\frac{f_x(x^*,y^*)}{f_y(x^*,y^*)} = \frac{g_x(x^*,y^*)}{g_y(x^*,y^*)}$$

Lagrange Analysis

- **Step 3: Solve the system of first-order conditions for *x*, *y*, and *λ***
  - **Often most interested in simply eliminating the multiplier…**
  - **From eqn 1), isolate *λ*:** $\lambda = -\frac{f_x(x,y)}{g_x(x,y)}$
  - **Insert expression for *λ* in eqn 2):** $f_y(x,y) - \frac{f_x(x,y)}{g_x(x,y)} g_y(x,y) = 0$
  - **Rearrange**
  - **Optimality condition: at the optimum (*x**, *y**)**

$$\frac{f_x(x^*, y^*)}{f_y(x^*, y^*)} = \frac{g_x(x^*, y^*)}{g_y(x^*, y^*)}$$

Graphical interpretation: at the constrained optimum, the function *f*(.) is tangent to the function *g*(.)



Lagrange Analysis

- **Apply Lagrange tools to consumer optimization**
- **Objective function: $u(c_1,c_2)$**
- **Constraint: $g(c_1,c_2) = Y - P_1c_1 - P_2c_2 = 0$**

- **Step 1: Construct Lagrange function**

$$L(c_1,c_2,\lambda) = u(c_1,c_2) + \lambda\left[Y - P_1c_1 - P_2c_2\right]$$

- **Step 2: Compute first-order conditions with respect to c_1, c_2, λ**

LAGRANGE ANALYSIS

- Apply Lagrange tools to consumer optimization
- Objective function: $u(c_1,c_2)$
- Constraint: $g(c_1,c_2) = Y - P_1c_1 - P_2c_2 = 0$

- **Step 1:** Construct Lagrange function

$$L(c_1,c_2,\lambda) = u(c_1,c_2) + \lambda\left[Y - P_1c_1 - P_2c_2\right]$$

- **Step 2:** Compute first-order conditions with respect to c_1, c_2, λ

1)

2)

3)

- **Step 3:** Solve (focus on eliminating multiplier from eqns 1 & 2)

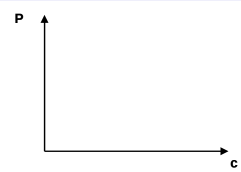
$$\frac{u_1(c_1^*,c_2^*)}{u_2(c_1^*,c_2^*)} = \frac{P_1}{P_2}$$

OPTIMALITY CONDITION

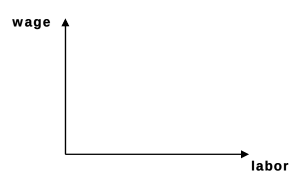
i.e., MRS = price ratio

THE THREE MACRO (AGGREGATE) MARKETS

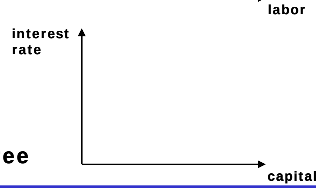
- Goods Markets



- Labor Markets



- Financial / Capital / Savings / Asset Markets



- Will put micro-foundations under all three