CONSUMPTION-SAVINGS FRAMEWORK

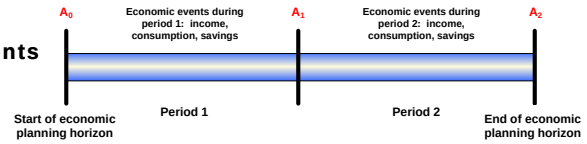
SEPTEMBER 19, 2011

BASICS

- Consumption-Savings Framework – provides foundation for
  - Goods-market demand function (again…but w/different interpretation)
  - Financial-market supply function
  - An application of basic consumer analysis…
  - …we will put a macro interpretation on it
  - Two time periods
    - Important: all analysis will be conducted from the perspective of the very beginning of period 1…
    - …so a "future" (period 2) for which to save
- Dynamic models are the staple of modern macroeconomic analysis
- An explicit accounting of time
- Two periods are sufficient to illustrate the basic principles
  - Soon will extend beyond two periods (Chapter 8)

BASICS

- **Timeline of events**



- **Notation**
  - c_1: consumption in period 1
  - c_2: consumption in period 2
  - P_1: nominal price of consumption in period 1
  - P_2: nominal price of consumption in period 2
  - Y_1: nominal income in period 1 ("falls from the sky")
  - Y_2: nominal income in period 2 ("falls from the sky")
  - A_0: nominal wealth at the beginning of period 1 / end of period 0
  - A_1: nominal wealth at the beginning of period 2 / end of period 1
  - A_2: nominal wealth at the beginning of period 3 / end of period 2
  - i: nominal interest rate between periods
  - r: real interest rate between periods
  - π_2: net inflation rate between period 1 and period 2 $\pi_2 = \frac{P_2 - P_1}{P_1}\left(= \frac{P_2}{P_1} - 1\right)$
  - y_1: real income in period 1 (= Y_1 / P_1)
  - y_2: real income in period 2 (= Y_2 / P_2)

STOCKS VS. FLOWS

- **Stock variables, aka accumulation variables**
  - Quantity variables whose natural measurement occurs at a **particular moment in time**

Economic examples:
- Checking account balance
- Credit card indebtedness
- Mortgage loan payoff

Interpret *A* in our framework as net wealth (= total assets – total debts)

- **Flow variables**

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Economic examples:
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- **Flow variables**
  - Quantity variables whose natural measurement occurs over the course of a **given interval of time**

Economic examples:
- Income
- Consumption
- Savings

- **The two broad categories of income**
  - Labor income
  - Asset income (generated by interest rate(s) on (components of) wealth)

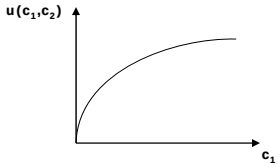
All income is a FLOW regardless of source

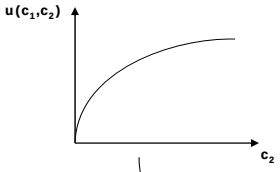
BASICS

- Building blocks of consumption-savings framework
- **Utility**
  - Describes the **benefits** of engaging in financial market (and other) activities
- **Budget constraint**
  - Describes the **costs** of engaging in financial market (and other) activities
- Utility and budgets two ***DISTINCT*** concepts
  - As in basic consumer analysis (Chapter 1)
- Only after describing utility and budgets separately do we bring the two together to obtain predictions from the framework

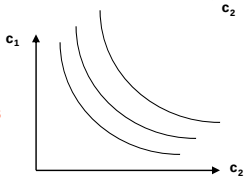
UTILITY

- Preferences $u(c_1, c_2)$ with all the "usual properties"
  - Lifetime utility function
  - Strictly increasing in c_1
  - Strictly increasing in c_2
  - Diminishing marginal utility in c_1
  - Diminishing marginal utility in c_2





- Plotted as indifference curves



- Utility side of consumption-savings framework identical to Chapter 1 framework

BUDGET CONSTRAINT(S)

- Suppose again Y "falls from the sky"
  - Y_1 in period 1, Y_2 in period 2
- Need two budget constraints to describe economic opportunities and possibilities
  - One for each period
  - Period-1 budget constraint

$$\underbrace{P_1 c_1 + A_1}_{\text{Total expenditure in period 1:}} = \underbrace{Y_1 + (1+i)A_0}_{\text{Total income in period 1:}}$$

Total expenditure in period 1: period-1 consumption + wealth to *carry into period* 2

Total income in period 1: period-1 Y + income from wealth *carried into period* 1 (inclusive of interest)

  - Period-2 budget constraint

$$\underbrace{P_2 c_2 + A_2}_{\text{Total expenditure in period 2:}} = \underbrace{Y_2 + (1+i)A_1}_{\text{Total income in period 2:}}$$

Total expenditure in period 2: period-2 consumption + wealth to *carry into period* 3

Total income in period 2: period-2 Y + income from wealth *carried into period* 2 (inclusive of interest)

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$$P_1c_1 + A_1 = Y_1 + (1+i)A_0 \quad \longleftrightarrow \text{can rewrite as} \quad P_1c_1 + A_1 - A_0 = Y_1 + iA_0$$

Savings during period 1 (a flow); Asset income during period 1 (a flow)

Total expenditure in period 1: period-1 consumption + wealth to *carry into period* 2

Total income in period 1: period-1 *Y* + income from wealth *carried into period* 1 (inclusive of interest)

  - Period-2 budget constraint

$$P_2c_2 + A_2 = Y_2 + (1+i)A_1 \quad \longleftrightarrow \text{can rewrite as} \quad P_2c_2 + A_2 - A_1 = Y_2 + iA_1$$

Savings during period 2 (a flow); Asset income during period 2 (a flow)

Total expenditure in period 2: period-2 consumption + wealth to *carry into period* 3

Total income in period 2: period-2 *Y* + income from wealth *carried into period* 2 (inclusive of interest)

DEFINITION: A consumer's **savings** during a given period is the ***change*** in **his wealth** during that period

BUDGET CONSTRAINT(S)

- Adopt a **lifetime** view of the budget constraint(s)
  - All analysis conducted from perspective of beginning of period 1
  - Period-1 budget constraint $P_1c_1 + A_1 = Y_1 + (1+i)A_0$
  - Period-2 budget constraint $P_2c_2 + A_2 = Y_2 + (1+i)A_1$

Asset position at end of period 1 / beginning of period 2 the key link - will think further about this soon...

Assume = 0 (no bankruptcies + strictly increasing utility)

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  - Combine into **lifetime budget constraint (LBC)**
    - Solve period-2 budget constraint for A_1...
    - ...and substitute into period-1 budget constraint

$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i} + (1+i)A_0$$

Present discounted value (PDV) of all lifetime expenditure

Present discounted value (PDV) of all lifetime income

For graphical simplicity, will often assume $A_0 = 0$ (i.e., consumer begins planning horizon with zero net wealth).
Note this is a *different* assumption than $A_2 = 0$.

LIFETIME BUDGET CONSTRAINT

$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

↓ subtract $P_1 c_1$

$$\frac{P_2 c_2}{1+i} = -P_1 c_1 + Y_1 + \frac{Y_2}{1+i}$$

↓ divide by P_2

$$\frac{c_2}{1+i} = -\left(\frac{P_1}{P_2}\right) c_1 + \frac{Y_1}{P_2} + \frac{1}{1+i}\frac{Y_2}{P_2}$$

↓ multiply by $(1+i)$

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right) c_1 + \frac{(1+i)Y_1}{P_2} + \frac{Y_2}{P_2}$$

LIFETIME BUDGET CONSTRAINT

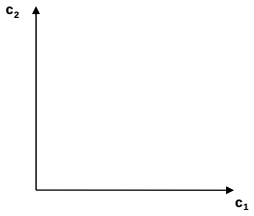
Graphically

$$P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

Solve for c_2

$$c_2 = -\left(\frac{P_1(1+i)}{P_2}\right) c_1 + \left(\frac{1+i}{P_2}\right) Y_1 + \frac{Y_2}{P_2}$$

Rearrange further using definition of inflation: $1+\pi_2 = \frac{P_2}{P_1} \Rightarrow \frac{1}{1+\pi_2} = \frac{P_1}{P_2}$



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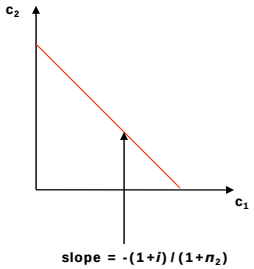


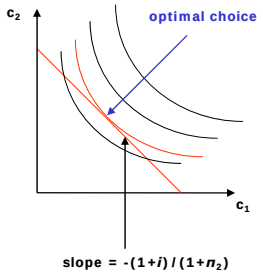
slope = $-(1+i)/(1+\pi_2)$

The larger is $(1+i)/(1+\pi_2)$, the steeper is the budget line

IMPORTANT: Changes in nominal interest rates (Fed) and/or inflation affect the slope of the LBC

CONSUMER OPTIMIZATION

- **Consumer's decision problem:** maximize lifetime utility subject to lifetime budget constraint – bring together both cost side and benefit side
  - Choose c_1 and c_2 subject to $P_1 c_1 + \frac{P_2 c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$
  - Plot budget line
  - Superimpose indifference map



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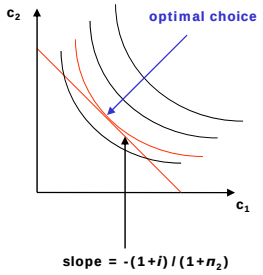
  - Plot budget line
  - Superimpose indifference map
- At the optimal choice



CONSUMPTION-SAVINGS OPTIMALITY CONDITION

- key result in modern macro analysis

$$\underbrace{\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)}}_{\text{MRS (between consumption in consecutive time periods)}} = \underbrace{\frac{1+i}{1+\pi_2}}_{\text{price ratio (across consecutive time periods)}}$$