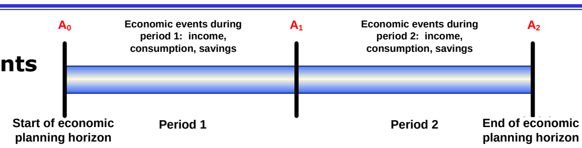
CONSUMPTION-SAVINGS FRAMEWORK (CONTINUED)

SEPTEMBER 21, 2011

BASICS

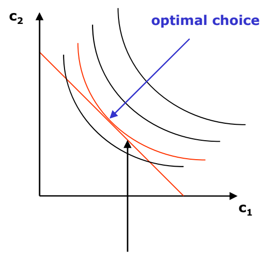
- **Timeline of events**



- **Notation**
  - c_1: consumption in period 1
  - c_2: consumption in period 2
  - P_1: nominal price of consumption in period 1
  - P_2: nominal price of consumption in period 2
  - Y_1: nominal income in period 1 ("falls from the sky")
  - Y_2: nominal income in period 2 ("falls from the sky")
  - A_0: nominal wealth at the beginning of period 1/end of period 0
  - A_1: nominal wealth at the beginning of period 2/end of period 1
  - A_2: nominal wealth at the beginning of period 3/end of period 2
  - i: nominal interest rate between periods
  - r: real interest rate between periods
  - π_2: net inflation rate between period 1 and period 2 $\pi_2 = \frac{P_2 - P_1}{P_1}\left(= \frac{P_2}{P_1} - 1\right)$
  - y_1: real income in period 1 ($= Y_1/P_1$)
  - y_2: real income in period 2 ($= Y_2/P_2$)

Consumer Optimization

- **Consumer's decision problem:** **maximize lifetime utility subject to lifetime budget constraint – bring together both cost side and benefit side**
  - **Choose c_1 and c_2 subject to** $P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$
  - **Plot budget line**
  - **Superimpose indifference map**



- **At the optimal choice**

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

- A key result in modern macro analysis

$$\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)} = \frac{1+i}{1+\pi_2}$$

MRS (between consumption in consecutive time periods) — **price ratio (across consecutive time periods)**

Derive consumption-savings optimality condition using Lagrange analysis

Lagrange Analysis

- **Apply Lagrange tools to consumption-savings optimization**
- **Objective function:** ***u*(c_1,c_2)**
- **Constraint:** $g(c_1, c_2) = Y_1 + \frac{Y_2}{1+i} - P_1c_1 - \frac{P_2c_2}{1+i} = 0$
- **Step 1:** **Construct Lagrange function**

$$L(c_1, c_2, \lambda) = u(c_1, c_2) + \lambda\left[Y_1 + \frac{Y_2}{1+i} - P_1c_1 - \frac{P_2c_2}{1+i}\right]$$

- **Step 2:** **Compute first-order conditions with respect to c_1, c_2, λ**

LAGRANGE ANALYSIS

- **Apply Lagrange tools to consumption-savings optimization**
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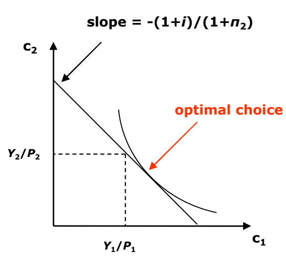
- **Step 2: Compute first-order conditions with respect to c_1, c_2, λ**
- **Step 3: Combine (1) and (2) (with focus on eliminating multiplier)**

$$\underbrace{\frac{u_1(c_1^*,c_2^*)}{u_2(c_1^*,c_2^*)}}_{\text{MRS (between consumption in consecutive time periods)}} = \underbrace{\frac{1+i}{1+\pi_2}}_{\text{price ratio (across consecutive time periods)}}$$

CONSUMPTION-SAVINGS OPTIMALITY CONDITION

SAVINGS AND ASSET POSITIONS

- **Definition: A consumer's savings during a given time period is the *change* in his wealth during that time period**
- **Assets/wealth (whether positive or negative) are a means for "transferring income over time"**



(continuing to assume $A_0 = 0$)

Optimal $c_1 > Y_1/P_1$ → consumer has negative wealth at end of period 1 → period-1 *savings* is negative (due to $A_0 = 0$)

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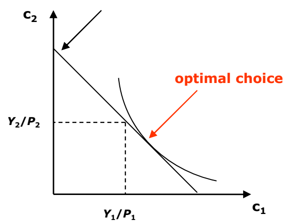

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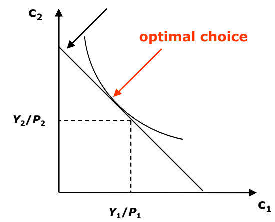
OR



Optimal $c_1 > Y_1/P_1$ → consumer has negative wealth at end of period 1 → period-1 *savings* is negative (due to A_0 = 0) } Consumer *dissaved* during period 1

Consumer *saved* during period 1 { Optimal $c_1 < Y_1/P_1$ → consumer has positive wealth at end of period 1 → period-1 *savings* is positive (due to A_0 = 0)

Fisher Equation

- **Nominal interest rate – measured in dollars**
- **Real interest rate – measured in goods**
- **Fisher equation: a link between the nominal interest rate, inflation rate, and real interest rate**
  - **"Strips out the effect of inflation"**
  - **Exact Fisher equation (will see foundations in Chapter 14)**

$$1+r=\frac{1+i}{1+\pi}$$

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More useful for our analytical framework

  - Approximate Fisher equation (intro macro)

$$(1+r)(1+\pi)=1+i$$

$$\cancel{1}+r+\pi+r\cancel{\pi}=\cancel{1}+i \quad (r\pi \approx 0)$$

In advanced economies, r and π are both generally small → $r\pi \approx 0$

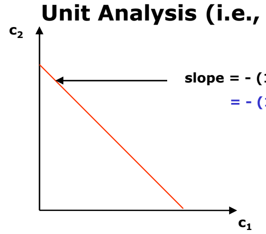
$$r=i-\pi$$

A useful rule of thumb

REAL INTEREST RATE

- r a key variable for macroeconomic analysis

- Unit Analysis (i.e., analyze algebraic units of variables)



slope = - $(1+i)/(1+\pi_2)$
= - $(1+r)$ BY FISHER EQUATION

Slope measures how much c_2 must be given up in order to obtain one more unit of c_1 ("rise over run") when saving or dissaving at market interest rates

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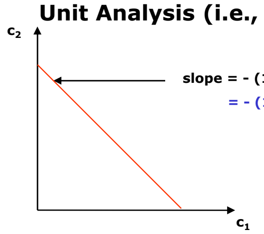

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Slope measures how much c_2 must be given up in order to obtain one more unit of c_1 ("rise over run") when saving or dissaving at market interest rates

$1+r$ is the *price* of period-1 consumption in terms of period-2 consumption

More generally: r measures the price of current goods in terms of future goods

- **Economic decisions depend on *real* interest rates (r), not nominal interest rates (i)**
  - Measures the cost of borrowing/lending in terms of goods…
  - …which is presumably what people most care about
- **Currently: nominal i (short-term) ≈ 0%, π ≈ 1.5% (CPI measure)**
  - Real interest rate < 0

TWO-PERIOD FRAMEWORK IN REAL TERMS

- **Depending on application, may be useful to work with framework (independent of lifetime vs. sequential approach) in nominal terms or in real terms**

$$P_1c_1 + \frac{P_2c_2}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

LBC in nominal terms (assuming $A_0 = 0$)

Divide by P_1

$$c_1 + \left(\frac{P_2}{P_1(1+i)}\right)c_2 = \frac{Y_1}{P_1} + \frac{Y_2}{P_1(1+i)}$$

Multiply *and* divide last term on right-hand-side by P_2

$$c_1 + \left(\frac{P_2}{P_1(1+i)}\right)c_2 = \frac{Y_1}{P_1} + \left(\frac{P_2}{P_1(1+i)}\right)\frac{Y_2}{P_2}$$

Use definitions: $y_1 = Y_1/P_1$, $y_2 = Y_2/P_2$, and $1+\pi_2 = P_2/P_1$

$$c_1 + \left(\frac{1+\pi_2}{1+i}\right)c_2 = y_1 + \left(\frac{1+\pi_2}{1+i}\right)y_2$$

Use Fisher equation: $(1+\pi_2)/(1+i) = 1/(1+r)$

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$$

LBC in real terms (assuming $A_0 = 0$)

Maximize $u(c_1,c_2)$ subject to the real LBC → identical consumption-savings optimality condition (details in recitations)

Consumption-Savings Optimality Condition

- **Emphasizing *i* and *π*** $$\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)} = \frac{1+i}{1+\pi}$$

  ↓ Fisher equation

- **Emphasizing *r*** $$\frac{u_1(c_1^*, c_2^*)}{u_2(c_1^*, c_2^*)} = 1+r$$

- **Can also analyze two-period framework sequentially, rather than from a lifetime perspective**

Lagrange Analysis: Sequential Approach

- **Sequential formulation highlights the role of net wealth (A_1) between period 1 and period 2**
  - **Accords better with the explicit timing of economic events than the lifetime approach…**
  - **…but yields the same result**
  - **Advantage: allows us to think about interaction between asset prices and macroeconomic events (intersection of finance theory and macro theory in Chapter 8)**

- **Apply Lagrange tools to consumption savings optimization**
- **Objective function: $u(c_1,c_2)$**
- **Constraints:**
  - **Period 1 budget constraint:** $Y_1 + (1+i)A_0 - P_1c_1 - A_1 = 0$
  - **Period 2 budget constraint:** $Y_2 + (1+i)A_1 - P_2c_2 - A_2 = 0$

  } TWO constraints

- **Sequential Lagrange formulation requires two multipliers**
  - **See Math Refresher, Chapter -1**