Economics 701
Advanced Macroeconomics I
Project 0 Suggested Solutions
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The numerical solutions for the six models:

Parameter Set	consumption	labor	capital	real wage	rental rate	lifetime utility
			SOCIAL PLANNING PROBLEM			
Baseline	0.9232	0.3000	14.5133	2.5846	0.0301	-558.33
High Depreciation	0.4595	0.3506	2.2322	1.2462	0.1101	-649.50
High Risk Aversion	0.9737	0.3169	15.3070	2.5846	0.0301	-82.85
			DECENTRALIZED EQUILIBRIUM			
Baseline	0.7570	0.2610	9.1965	2.3072	0.0368	-568.20
High Depreciation	0.4006	0.3074	1.7840	1.2053	0.1168	-652.66
High Risk Aversion	0.9114	0.3142	11.0716	2.3072	0.0368	-90.49

Steady-state lifetime utility (displayed in the last column) is computed as

$$\sum_{t=0}^{\infty} \beta^t u(c^{ss}, n^{ss}) = \frac{u(c^{ss}, n^{ss})}{1-\beta}$$

(recall the formula for an infinite summation of constant terms). Of course, the numerical values of utility themselves are meaningless because utility only provides an ordinal, not a cardinal, measure.

In computing the Social Planning solutions, you could have exploited the fact that the Social Planner would consider the labor income and capital income tax rates to each be zero, allowing you to write just one set of programs in which you simply set $\tau^n = \tau^k = 0$ to implement the Social Planning solution and $\tau^n > 0$ and/or $\tau^k > 0$ to compute the decentralized equilibrium solution. Also, note that one can compute the implied real wage and real rental rate of capital for the Social Planning solution (displayed in the upper panel of the table above), even though you were not asked to do so – these factor prices are simply the ones that support the efficient allocation.

For a given parameter set, lifetime utility (welfare) is obviously lower in the decentralized economy than under the Pareto efficient (i.e., the Social Planning) solution. This is of course to be expected because of the presence of the proportional taxes $\tau^n > 0$ and $\tau^k > 0$. Indeed,

the terminology "distortionary" taxation (which is what the positive labor- and capital-income tax rates here create) is meant to evoke the idea that the allocation achieved is not as "good" (in a utility sense) as the fully-efficient (Social Planning) allocation.[1] Computing lifetime utility under the efficient and decentralized solutions would have been a good diagnostic on your code – if your numerical results showed utility to be higher in the decentralized economy with positive taxes, clearly something went wrong because theory tells us this just cannot occur. "Pure" theory is often a good "sensibility check" on computational results.

For a given parameter set, steady-state consumption, labor, and capital are all lower in the decentralized equilibrium than in the efficient allocation. This is because taxation here causes a depressing effect on market-based economic activity.[2] Indeed, total GDP is lower in the decentralized economy than in the efficient allocation for each parameter set.

In moving from the baseline economy to the high-depreciation economy, steady-state consumption and capital both fall and steady-state equilibrium labor rises (whether considering the efficient solution or the equilibrium solution). The intuition behind these results is straightforward: with a rapid rate of capital obsolescence (i.e., machines wear out very quickly), it is more difficult to keep as many machines around. Because in the steady state, the capital stock is used to feed consumption, a lower capital stock is associated with a lower rate of consumption. Maintaining the same level of capital in the high-depreciation economy as in the baseline economy would require consumers to lower consumption by *even more* than they currently already have to.

In moving from the baseline economy to the high risk-aversion economy, the steady-state capital stock rises (whether considering the efficient solution or the equilibrium solution) as consumers accumulate more total savings – associated with this is a rise in the level of steady-state consumption. But this is only a level effect, one that could be undone with an appropriate re-scaling of the steady-state *level* of TFP (whose normalization to unity is arbitrary anyway – and **if** the parameterization was $\sigma = 1$). Indeed, if you also compute steady-state ratios of consumption/GDP and capital/GDP, shown in the table below, you'll find that they are *invariant* to the parameter σ (again, if we had $\sigma = 1$). Often what one is concerned with is not a model's pure levels of steady-state variables, but rather a model's "great ratios" (i.e., consumption/GDP and capital/GDP). The fact that these are invariant to σ in the steady-state stems from the fact that the steady-state capital output ratio is determined exclusively by the rate of time preference and the two technology parameters α and δ,

[1] Note, however, that proportional taxes in and of themselves are not distortionary – that is, "proportional" and "distortionary" are *not* synonyms. The canonical undergraduate introductory microeconomics example of a tax that is proportional but not distortionary is one that is levied on a good whose supply is perfectly inelastic.

[2] Note well the qualification "market-based" here. Steady-state *leisure* of course is higher in the decentralized economy for any given parameter constellation. In models that identify some type of "home-based economic activity" (notably, the "home-production" models of Greenwood, Rogerson, and Wright (1995) and others), home-based economic activity would rise in the presence of labor- and capital-income taxation. The ideas behind such a result is simple: labor- and capital-income taxation as we typically think of them tax *only* market-based economic activity, so the economy substitutes away from market-based production and towards home production (such as, child rearing, cooking at home, etc). See the Greenwood, Rogerson, and Wright chapter in *Frontiers of Business Cycle Research* and the references therein for more on home-production models.

independent from σ (because σ drops out in the steady-state version of the intertemporal Euler equation) – see our discussion of calibration of the basic RBC model for more on this.

However, these comparative statics on the parameter σ are just computational exercises, with little (or no) meaning. This is because given additive separability in preferences between consumption and labor, we need $\sigma = 1$ to satisfy balanced growth restrictions (if we think those are natural ones for the underlying **growth** model to satisfy; absent compelling reason and model-based justification for abandoning them, we should our model(s) to display balanced growth).

Parameter Set	**GDP**	**consumption/GDP**	**capital/GDP**
		SOCIAL PLANNING PROBLEM	
Baseline	1.2135	0.7608	11.9597
High Depreciation	0.6827	0.6730	3.2697
High Risk Aversion	1.2799	0.7608	11.9597
		DECENTRALIZED EQUILIBRIUM	
Baseline	0.9410	0.8045	9.7733
High Depreciation	0.5790	0.6919	3.0813
High Risk Aversion	1.1328	0.8045	9.7733

Finally, a somewhat "philosophical" note is that comparing utility across parameter sets may not be very meaningful because each parameter set could essentially be thought of as a "completely different world" (at least this is how many in the profession think of it). The maintained assumption in virtually all macro models is that parameters such as the depreciation rate, the coefficient of relative risk aversion, etc. are immutable truths about the world and, in particular, are unable to be affected by policy choices. As such, it is perhaps a purely philosophical question to ask whether people would be "better off" if the depreciation rate (an immutable truth about the world, at least in our DSGE models…) were higher or lower, or if people would be "better off" if they weren't so risk averse, etc. Because policy simply cannot change these "truths" about the world, there perhaps is not much interest or purpose in asking such questions in a model that cannot or does not allow considering such issues.[3]

[3] Of course, one might plausibly protest that things like σ and δ are **not** immutable truths about the world, but rather themselves perhaps amenable to purposeful action on the part of consumers, firms, and the government. If one adopts this view, asking such questions then would be meaningful. Of course, one would then need to take a stand on the mechanisms by which purposeful action on the part of various economic actors influences such things – i.e., one would need to model how government policy, for example, affects consumers' degree of risk aversion. Some deep and interesting questions, for sure…but ones that macroeconomics by and large so far has not/does not tackle. Thus, in the context of our DSGE models, we will invariably treat objects such as σ, δ, etc. as "immutable truths."