Department of Economics University of Maryland

Economics 701
Advanced Macroeconomics I
Project 1
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Objective

As a stepping stone to learning how to work with and computationally solve novel and medium-scale or large-scale DSGE models (should your research interests eventually take you in that direction), you will compute a first-order approximation to the decision rules of an RBC economy using the Schmitt-Grohe and Uribe (2004 *Journal of Economic Dynamics and Control*) algorithm. Because the primary objective here is **to learn how to implement** such solutions yourself, you are not permitted to use off-the-shelf programs provided by Schmitt-Grohe and Uribe or others or packaged programs such as Dynare.

The Problem

For the Social Planning problem of the RBC economy with long-run growth, construct a linear approximation of the model's (dynamic) decision rules around the deterministic steady state. Specifically, in terms of the notation of Schmitt-Grohe and Uribe (2004), you must solve (using Matlab or another program such as Fortran, etc) the system of equations

$$f_{y'} \cdot g_x \cdot h_x + f_y \cdot g_x + f_{x'} \cdot h_x + f_x = 0$$

for the **matrices** g_x and h_x. Keep in mind that each term of this expression is evaluated at the deterministic steady state of the model.

The Social Planning problem for the **transformed RBC economy with possibly time-varying long-run growth** is: maximize the representative household's lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} b^t X_t^{1-\sigma} u(c_t, n_t)$$

subject to the sequence of resource constraints

$$c_t + \gamma_t k_{t+1} - (1-\delta) k_t = z_t f(k_t, n_t),$$

taking as given the exogenous law of motion governing (high-frequency) TFP fluctuations z_t ,

$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon^z_{t+1},$$

and the exogenous law of motion governing (low-frequency) trend growth fluctuations,

$$\ln \gamma_{t+1} = (1-\rho_\gamma)\ln \bar{\gamma} + \rho_\gamma \ln \gamma_t + \varepsilon^\gamma_{t+1},$$

which in turn governs the evolution of the deterministic component of productivity

$$X_{t+1} = \gamma_t X_t .$$

The stationary transformation of the (stochastically) growing economy is taken with respect to the "very-long-run growth rate" $\bar{\gamma}$.

The steady-state level of TFP is $\bar{z} = 1$, with innovations to TFP $\varepsilon^z_t \sim \text{iid } N(0,\sigma^2_z)$. The steady-state level of long-run productivity **growth** (in gross terms) is $\bar{\gamma}$ (i.e., the "very-long-run growth rate"), with innovations to long-run growth $\varepsilon^\gamma_t \sim \text{iid } N(0,\sigma^2_\gamma)$.

The parameter *b* is the (constant) subjective discount factor in the underlying growing economy, and the utility parameter σ is introduced below.

The deterministic component of productivity grows at a stochastic (gross) growth rate γ_t between period *t* and period *t*+1. The growth rate γ_t is revealed to agents in the economy at the same time as z_t is revealed. Thus, agents are (somewhat) informed (although not perfectly informed) about how outcomes (realized prices and quantities) will fluctuate between period *t* and period *t*+1, which is the essence of deterministic growth.

Functional forms for period-t utility and production are

$$u(c_t, n_t) = \frac{\left[c_t(1-n_t)^\psi\right]^{1-\sigma} - 1}{1-\sigma}$$

and

$$f(k_t, n_t) = k_t^\alpha n_t^{1-\alpha} .$$

Finally, government absorption and taxes are always zero.

Quantitatively analyze this RBC economy with TFP and long-run growth shocks for the three parameter sets shown below:

	Parameter Set A (Baseline)	**Parameter Set B (Constant Growth)**	**Parameter Set C (Variable Growth)**
$\bar{\gamma}$	1	$1.03^{1/4}$	$1.03^{1/4}$
b	0.99	0.99	0.99
δ	0.02	0.02	0.02
σ	1.5	1.5	1.5
α	0.36	0.36	0.36
ψ	**???**	**???**	**???**
ρ_z	0.92	0.92	0.92
σ_z	0.006	0.006	0.006
ρ_γ	---	---	0; 0.5
σ_γ	---	---	0.001; 0.003

Note the "**???**" listed for the parameter ψ in all three parameter sets. In each case, you must choose the value of ψ that makes the steady-state fraction of time spent in employment = 0.3 of the total (unit) time endowment..

Simulations

Having computed the matrices g_x and h_x, the next step is to conduct simulations of your model(s). In order to generate simulations, recall that the first-order approximations are given by

$$y_t = g(x_t, \sigma) \approx g(\bar{x}, 0) + g_x \cdot (x_t - \bar{x})$$
$$x_{t+1} = h(x_t, \sigma) \approx h(\bar{x}, 0) + h_x \cdot (x_t - \bar{x}) + \eta\sigma\varepsilon_{t+1}$$

in which it is easiest to set the perturbation parameter σ = 1, in which case the matrix η must contain the exogenous standard deviations of the model's state variables. You will be provided with sequences of shocks for the vector process $\{z_t, \gamma_t\}$ which will be the forcing process for your time-series simulations. Specifically, you will be provided with 200 sequences each of length 200 periods (quarters). These shocks are drawn from an *iid* $N(0, 1)$ distribution, which, when pre-multiplied with the appropriate row of η yields an *iid* $N(0, \sigma_i^2)$ sequence, $i \in \{z, \gamma\}$.[1]

For parameter set C, you should determine some logical and informative sets of experiments that help you and the reader understand the joint effects of low-frequency and high-frequency productivity fluctuations.

[1] The shocks were generated using Matlab's built-in `randn` function. For this project, use the provided sequence of shocks for your simulations (for the sake of some comparability). In subsequent projects of your own, you can use the `randn` function to generate your own random numbers.

Using the **HP-filtered cyclical components of your simulated time series** (specifically, the net percentage deviation of each simulated series from its respective HP-filtered trend), calculate, for each time series of interest in a given simulation, standard deviations, first-order serial correlations, and contemporaneous correlation of each variable with GDP.[2,3] Then, compute the medians of these means, the medians of these standard deviations, and the medians of these correlations across all simulations. These latter three sets of second-moment statistics (along with the steady-state values of the endogenous variables you decide are interesting/relevant to analyze) are what you should report as your simulation-based results (in some appropriate and informative combination of tables and/or graphs and/or text).

Analysis/Discussion

Compare your results with appropriate empirical data (either collected and summarized yourself or referencing existing some existing study/studies – for example, King and Rebelo (1999), Cooley and Hansen (1995), or some other existing and credible study). These comparisons are likely best made through some informative collection of tables and/or graphs and/or text. The exact data that you compare your model to is left up to you – this should also be reflected in how you motivate your paper in the introduction and abstract.

The discussion of results is in many ways the most important part of your paper. Here, you should provide interesting and relevant analysis from the (informative) experiments you run, describing the successes as well as shortcomings of your model. Describe the intuition/economic mechanism for any major successes; discuss the intuition/economic mechanism for any important shortcomings. Your discussion need not describe every nitty-gritty detail of the results you obtain (and should certainly NOT be just a verbal description of what a reader could find in tables, etc.), but should provide a fair and scientific view of your results and how they do or do not shed light on the study's basic hypotheses and goals.

Two issues/questions that you must analyze are:

1. Analytical and/or quantitative exploration of the Frisch elasticity of labor supply in the model.
2. Explore sensitivity with respect to the persistence of trend-growth fluctuations, and standard deviation of shocks to trend-growth fluctuations.

(Some) Computational/Programming Guidance

Using Matlab's `fsolve` function to solve for the matrices g_x and h_x is once again the key computational step, as in Project 0.

[2] Note that some series (such as GDP) may have to be constructed residually if you do not include them as part of your state or costate vectors.
[3] You will be provided with two Matlab files that implement the HP filter.

In order to conduct simulations using the sequences of shocks with which you will be provided, you must essentially proceed "iteratively" through each simulation. To do so, begin with k_0 (which is simply the deterministic steady-state value $\bar{k}$) and the "first realization" of the shocks to z and γ (that is, the first (period-zero) shocks to log z and log γ) and compute the period-zero equilibrium outcome using

$$y_0 = g(\bar{x},0) + g_x \cdot (x_0 - \bar{x})$$
$$x_1 = h(\bar{x},0) + h_x \cdot (x_0 - \bar{x}) + \eta\sigma\varepsilon_1$$

Once you have the period-zero equilibrium outcome of the model in hand, compute the period-one equilibrium outcome of the model using

$$y_1 = g(\bar{x},0) + g_x \cdot (x_1 - \bar{x})$$
$$x_2 = h(\bar{x},0) + h_x \cdot (x_1 - \bar{x}) + \eta\sigma\varepsilon_2$$

Continue this way through all periods of the simulation, and then repeat this for each of the simulations. In conducting these simulations, you can and should try to cleverly arrange matrices and vectors in a way that takes advantage of Matlab's comparative advantage (compared to other software programs) in performing matrix manipulations. Be careful about issues such as matrix conformability, in particular with your g_x and h_x matrices.

A "sensibility check" you may want to try on your programs is to check the convergence (to the deterministic steady state) implied by your computed g_x and h_x matrices. To check this, begin with some arbitrary k_0 (say, perhaps 1% above or below the steady-state $\bar{k}$) and construct a vector of zeros for the sequence of TFP shocks and deterministic productivity shocks. Iteratively apply your approximated decision rules (as described above) to construct a time-series simulation of the model – the difference, of course, is that this will be a *deterministic simulation* because each period the TFP shock and the trend shock is by assumption zero. If you have computed the correct g_x and h_x, your model variables should clearly converge to their deterministic steady-state counterparts. If you do not find convergence to the deterministic steady state (and you are convinced you are conducting the simulations correctly), there likely is an error in your computed g_x and/or h_x matrix.

Stand-Alone Section

After the model section, everyone should submit a compact section that clearly defines and runs the **following experiments for Parameter Set A, Parameter Set B, and Parameter Set C (with zero shocks to the growth rate of the long-run component of productivity, and the low variance (σ_γ = 0.001) of the long-run component of productivity):**

1. Solve for the steady state, and **report the steady state values of *c*, *n*, *k*, *c*/*y*, *k*/*y*, the real interest rate, and the real wage.** Provide brief analysis/discussion, within a parameter set, and across parameter sets. **Be clear that a description of the numerical results is *not* analysis/discussion; interpret somehow the results.**
2. Simulate each economy, and report the King and Plosser (1999) model-based results for ***c*, *n*, *k*, *c*/*y*, *k*/*y*, the real interest rate, and the real wage.** Provide brief analysis/discussion, within a parameter set, and across parameter sets. **Be clear that a description of the numerical results is *not* analysis/discussion; interpret somehow the results.**

What Else to Submit

Following the above stand-alone section, conduct whatever other experiments you wish to. However, all of the experiments, etc. should be conducted with a view towards understanding how the model "works" (where "works" should mean what you can learn from the sets of experiments conducted, even if it is a repeat of things that show up in the literature – however, completely extraneous experiments will be discounted).

What To Submit

Your submission should be a stand-alone, complete paper – i.e., one should be able to read it independent of knowing what the "description" of "Project 1" was. As before, your submission **must be typed, not hand-written.**

The overall structure of the paper must include:

- Abstract
- Introduction/Motivation
- Precise empirical methodology (if constructing your own empirical statistics)
- Model details, with (perhaps in an Appendix) important derivations and efficiency conditions, along with intuition/discussion of the important parts of the structure of the model
- **A clear definition of the solution of the model**
- Solution method for the model, including
    - A clear description of how you solve for the parameter set ψ, with its numerical values reported
    - Presenting your computed g_x and h_x matrices (i.e., report the numerical values of these matrices)
- Description of the experiments you perform, and why
- Results from informative experiments, including comparison with data and appropriate discussion and/or interpretation
- Conclusions, summarizing your results, their potential importance, and suggestions, if any, for future work

As for Project 0, attach a print-out of your code to your submission.