Does the Timing of the Cash-in-Advance Constraint Matter for Optimal Fiscal and Monetary Policy?

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Abstract

We quantitatively demonstrate that the precise timing of financial markets and goods markets in a flexible-price cash good/credit good model does not matter for the main baseline results in the Ramsey literature on optimal fiscal and monetary policy. This result is reassuring because Ramsey analysis, in the tradition begun by Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991), has been applied to a quickly-expanding rich class of DGSE models recently, making it important to know whether models based on these original structures have been pursuing a mirage. In the original models, the timing is such that nominal money holdings are freely-adjustable in response to shocks in the period in which they will be used to purchase consumption. We alter this timing convention so that nominal balances cannot be adjusted in the period they will be used — the timing assumption of Svensson (1985) — to study how sensitive the baseline results are to this slight, and ultimately ad-hoc, modification. Our broad finding is that Ramsey-optimal inflation continues to display very high variability just as in the original models, although this can differ depending on exactly which exogenous processes are driving the economy. The basic intuition for the result is that, no matter the timing of markets, inflation variability creates no relative price distortions. Thus, interpretation of results from the recent spate of Ramsey studies, which have had as a primary motivation the determination of the optimal degree of inflation stabilization in the presence of various features and frictions in the economy, is not blurred by the choice of cash/credit timing.

Keywords: cash-in-advance, Ramsey models
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1 Introduction

Ramsey analysis of optimal fiscal and monetary policy, in the tradition begun by Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991), has been applied to a quickly-expanding rich class of DGSE models recently. A major goal of these recent efforts has been to quantitatively determine the optimal degree of inflation stabilization. One strand of this resurgent literature has employed as its core the cash-good/credit-good structure *and timing* of Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991), in which a central result is that the optimal rate of inflation variability is extremely high. Because the timing of these original models is such that nominal money holdings are freely-adjustable in response to shocks in the period in which they will be used to purchase consumption, a natural concern is whether the benchmark results in this literature may be sensitive to the precise timing of markets. If the results are indeed sensitive to this ultimately ad-hoc assumption, subsequent results obtained in richer models become difficult to interpret. To assess this issue, we incorporate the timing of Svensson (1985), in which nominal balances are pre-determined in the period they will be used to purchase consumption, into an otherwise-unaltered basic cash/credit environment. Our main result is that optimal inflation variability continues to be very high in the face of supply shocks, but not in the face of pure fiscal shocks. Because changing the timing of markets does not completely eliminate the basic Ramsey model's bias towards inflation volatility, we conclude that concerns about cash/credit-based Ramsey models along the timing dimension are not critical.

The reason behind optimal inflation volatility in basic Ramsey monetary models is well-understood. In a coordinated program of fiscal and monetary policy, the Ramsey planner prefers state-contingent movements in the price level to changes in proportional taxes in response to any shock that affects the consolidated government budget in equilibrium. In essence, the Ramsey government wraps an optimization problem around the fiscal theory of the price level (FTPL), trading off the welfare consequences of variations in the price level against the welfare consequences of variations in primary fiscal surpluses. The result was first quantitatively demonstrated by Chari, Christiano, and Kehoe (1991), who adopted the cash/credit structure and timing of Lucas and Stokey (1983) — hereafter, CCK and LS, respectively. After lying dormant for nearly a decade, Ramsey models of jointly-optimal fiscal and monetary policy (both those based on the LS/CCK cash/credit structure as well as others that motivate money demand by other means) have enjoyed a resurgence. Models that have employed the LS/CCK structure and timing as their core are those of Siu (2004), Chugh (2006a, 2006b), and Arseneau and Chugh (2007); some of these studies continue to find high inflation variability, while some reach the opposite conclusion. As Ramsey models grow ever richer, it is important to know whether the benchmark against which new results are compared is robust to a relatively ad-hoc timing convention.[1]

The crux of the issue surrounds the cash-in-advance constraint imposed on a subset of goods (dubbed "cash goods" and denoted by c_1 in what follows), which, when binding in equilibrium, reads

$$P_t c_{1t} = M_t \tag{1}$$

regardless of whether one employs the LS/CCK timing or the Svensson (1985) timing. Here, M_t is the representative household's nominal holdings of money to be used for period-t cash purchases, and P_t denotes the nominal price of cash goods (equivalently, the nominal price level of the economy). Under LS/CCK timing, M_t is free to be chosen by households in response to the period-t state of the economy — that is, M_t can be adjusted before the purchase of cash goods. In contrast, under Svensson (1985) timing, M_t is pre-determined in period t — it cannot be adjusted in response to the realized period-t state of the economy. The interpretation of these two timing conventions is that financial markets meet before goods markets in the LS/CCK environment, whereas the opposite occurs in the Svensson (1985) environment. A natural conjecture is that this difference in the timing of markets may render the welfare consequences of state-contingent variations in P_t — which is what Ramsey inflation variability is all about — in the two environments quite different because they would imply quite different dynamics for consumption.

Our results show this conjecture overall has little quantitative bite. Regardless of whether we use LS/CCK timing or Svensson (1985) timing, Ramsey inflation variability is very high when the environment is buffeted by both TFP shocks and pure government absorption shocks. Ramsey inflation variability acts as a lump-sum tax on the nominal government debt pre-accumulated by households, and this channel of optimal policy has little to do with the timing of markets. Based on the results of incarnations of Ramsey models that feature nominal rigidities in straightforward ways (as in Siu (2004), Schmitt-Grohe and Uribe (2004b), and Chugh (2006a)) as well the results of Aruoba and Chugh (2006) (in which deep-rooted frictions underlying monetary exchange provide a novel force shaping optimal policy), the effect needed to overturn the basic Ramsey prescription of price level volatility is within-period relative price distortions induced by actual inflation. Such an effect is not induced by a change in the timing of markets.

However, we demonstrate that the degree of optimal inflation variability *is* very sensitive to timing assumption when it is only pure government absorption shocks that drive the economy. In the LS/CCK environment, Ramsey inflation is volatile in the face of pure government purchase shocks, although much less volatile than in the face of TFP shocks. In contrast, in the Svensson (1985)

[1] Ramsey models that employ something other than a cash/credit structure to motivate money demand — notably, the sequence of studies by Schmitt-Grohe and Uribe (2004a, 2004b, 2005) that use a velocity-based transactions-cost motive — are immune to the issue we investigate here.

environment, we find that Ramsey inflation does not respond at all to pure government purchase shocks. This latter result shows that the conjecture that Svensson (1985) timing may change some basic Ramsey prescriptions is indeed correct. Our result that basic Ramsey prescriptions are not overturned in the presence of a more complete set of driving processes, though, leads us to conclude that the finding of a zero inflation response to government purchase shocks is a very particular one, one that does not damn the basic results in the literature.

Our investigation is about some of the primitives of simple Ramsey models of fiscal and monetary policy. Some, likely even many, researchers and policy-makers are reluctant to take literally the basic Ramsey prescription that inflation policy should be used to finance shocks that affect the government budget. Even with such a view, one with which we are sympathetic, the recent results in the Ramsey literature are informative because they have cast some light on what features or frictions in the economy may make pursuing inflation stability important as well as what features or frictions in the economy may *not* be important for recommending inflation stability as an overriding goal of monetary policy. Actual economies are buffeted by both demand and supply shocks, and (monetary) policy-makers typically think it is optimal to stabilize inflation no matter the forces driving business cycles. The Ramsey framework can thus be usefully viewed as a quantitative laboratory for isolating features of an economy important for recommending inflation stability, something about which New Keynesian models, the workhorse class of models used for the analysis of optimal monetary policy, are silent because they effectively just assume inflation stability is important. Because it thus can serve as a diagnostic tool, it is important to fully understand how the basic Ramsey model operates; our work here contributes to this.

The rest of our analysis is organized as follows. Section 2 sketches the basic LS/CCK environment and Ramsey problem, and Section 3 presents the same environment and the Ramsey problem with a Svensson (1985)-based timing. In Section 4, we quantitatively characterize properties of optimal policy in the two environments, and the main result is that Ramsey inflation variability is very high in both in the face of a complete set of exogenous shocks. In Section 5, we discuss further the intuition behind why, from the perspective of the Ramsey planner, inflation variability is desirable no matter the timing of markets, and Section 6 concludes.

2 LS/CCK Timing

The basic environment is presented in detail in the original LS and CCK studies and recapped in Chari and Kehoe (1999) as well as the subsequent studies by Siu (2004), Chugh (2006a, 2006b), and Arseneau and Chugh (2007). We thus only sketch the basic structure needed for us to analyze the issue at hand.

2.1 Government

There is a consolidated fiscal-monetary authority that finances exogenous government expenditures through a combination of proportional labor income taxes, nominal one-period debt, and money creation. The government's flow budget constraint is

$$M_t + B_t + \tau^n_{t-1} P_{t-1} w_{t-1} n_{t-1} = P_{t-1} g_{t-1} + R_{t-1} B_{t-1} + M_{t-1}, \tag{2}$$

The left-hand-side is the government's receipts in period t, which consist of issuance of new money M_t, issuance of new nominal debt B_t, and labor tax revenue from the previous period $\tau^n_{t-1} P_{t-1} w_{t-1} n_{t-1}$, where τ^n_t is the flat tax rate on labor, w_t is the real wage, and n_t is equilibrium labor. The right-hand-side is government outlays in period t, which consist of paying for its nominal purchases the previous period $P_{t-1} g_{t-1}$, nominal debt repayment, and retiring of the outstanding money stock.

2.2 Firms

Firms are simple: the representative firm operates the linear-in-labor production technology $f(n_t, z_t) = z_t n_t$, where z_t is the exogenous level of TFP. Firm profit-maximization implies $w_t = z_t$.

2.3 Households

There is a measure-one continuum of identical, infinitely-lived households. The representative household trades in the following way. At the beginning of period t, it holds nominal government bonds B_{t-1} and money M_{t-1}. All uncertainty is resolved before any trading occurs in period t. It first engages in financial market trading. The government's money injection happens in the financial market, in the form of open-market operations. The household also receives debt repayment, inclusive of interest, $R_{t-1} B_{t-1}$, receives its after-tax labor income earned the previous period, $(1 - \tau^n_{t-1}) w_{t-1} n_{t-1}$, and settles its bill for consumption purchases from the previous period, $P_{t-1} c_{1t-1} + P_{t-1} c_{2t-1}$. The household then reallocates its nominal portfolio by choosing M_t and B_t with which it exits the financial market. It then proceeds to the goods market, in which it sells labor n_t and buys cash goods and credit goods. This consumption will not be paid for until the financial market in period $t + 1$, and labor income will not be received until period $t + 1$. The constraints facing the household in period t are thus

$$M_t - M_{t-1} + B_t - R_{t-1} B_{t-1} = (1 - \tau^n_{t-1}) P_{t-1} w_{t-1} n_{t-1} - P_{t-1} c_{1t-1} - P_{t-1} c_{2t-1} \tag{3}$$

and the cash-in-advance constraint

$$P_t c_{1t} \leq M_t. \tag{4}$$

The first constraint combines the household's trading in the financial market and the goods market, while the second constraint requires that a subset of consumption goods ("cash goods") be purchased with money. Credit goods, c_2, are exempt from this cash requirement. In the cash-in-advance constraint, the period-t choice of money holdings is available for use in period t, because the optimal choice of M_t occurs in the first sub-period of period t. This timing assumption is the LS/CCK protocol. Finally, note that cash goods and credit goods have a relative price of one because they are assumed to be technologically identical.

The household chooses processes for c_{1t}, c_{2t}, n_t, M_t, and B_t to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, n_t) \tag{5}$$

subject to (3) and (4). Associate the Lagrange multipliers ϕ_t/P_{t-1} with the sequence of budget constraints and λ_t/P_{t-1} with the sequence of cash-in-advance constraints. The household's first-order conditions with respect to cash good consumption, credit good consumption, labor, money holdings, and bond holdings are thus:

$$u_{1t} - \lambda_t - \beta E_t \phi_{t+1} = 0, \tag{6}$$

$$u_{2t} - \beta E_t \phi_{t+1} = 0, \tag{7}$$

$$u_{3t} + (1 - \tau_t^n) w_t \beta E_t \phi_{t+1} = 0, \tag{8}$$

$$-\frac{\phi_t}{P_{t-1}} + \frac{\lambda_t}{P_t} + \beta E_t \left(\frac{\phi_{t+1}}{P_t} \right) = 0, \tag{9}$$

$$-\frac{\phi_t}{P_{t-1}} + \beta E_t \left(R_t \frac{\phi_{t+1}}{P_t} \right) = 0, \tag{10}$$

where u_{it} denotes the derivative of u with respect to the ith argument, evaluated at time-t arguments.

Combining (7) and (8) gives a standard consumption-leisure optimality condition,

$$-\frac{u_{3t}}{u_{2t}} = (1 - \tau_t^n) w_t. \tag{11}$$

From (10), we get a usual Fisher relation,

$$1 = R_t E_t \left[\frac{\beta \phi_{t+1}}{\phi_t} \frac{1}{\pi_t} \right], \tag{12}$$

where $\pi_t \equiv P_t/P_{t-1}$ is the gross rate of price inflation between period $t-1$ and period t. The stochastic discount factor $E_t[(\beta\phi_{t+1}/\phi_t)(1/\pi_t)]$ prices a nominally risk-free one-period asset. We can express the Fisher relation alternatively in terms of marginal utilities. Combining (6) and (9), we get

$$\phi_t = \frac{u_{1t}}{\pi_t}. \tag{13}$$

Substituting this expression into (12) gives us the pricing formula for a one-period risk-free nominal bond,

$$1 = R_t E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right]. \tag{14}$$

This condition (lagged one period) will be the condition that pins down the Ramsey-optimal inflation rate in the LS/CCK environment.

Using (14), the consumer first-order conditions also imply that the gross nominal interest rate equals the static marginal rate of substitution between cash and credit goods. Specifically,

$$R_t = \frac{u_{1t}}{u_{2t}}. \tag{15}$$

Note that in a monetary equilibrium, $R_t \geq 1$, otherwise consumers could earn unbounded profits by buying money and selling bonds.

2.3.1 Ramsey Allocations

For brevity and because interested readers can find it in LS and CCK, we skip the formal definition of a private-sector equilibrium and proceed straight to the Ramsey problem. The Ramsey problem is to maximize the expected lifetime utility (5) of the representative household subject to the resource constraint

$$c_{1t} + c_{2t} + g_t = z_t n_t, \tag{16}$$

which, note, has a unit marginal rate of transformation between cash goods and credit goods, along with all the equilibrium conditions of the economy. Note that government purchases act as a pure parallel shifter of the economy's resource frontier.

In principle, the equilibrium conditions include the potentially occasionally-binding constraint $R_t \geq 1$, but, by adopting the same assumption as CCK of weak separability of $u(.)$ between (c_1, c_2) and n, we can ignore this constraint in the formulation of the Ramsey problem. Indeed, CCK prove that with this assumption, $R_t = 1\ \forall t$ — that is, by implementing the Friedman Rule, the Ramsey planner always ensures no distortion along the cash/credit margin.

Apart from the resource constraint, then, the equilibrium conditions of the economy can be condensed into a single present-value implementability constraint (PVIC) (the derivation of which is omitted because it can be found in CCK)

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{1t} c_{1t} + u_{2t} c_{2t} + u_{3t} n_t \right] = A_0^{CCK}, \tag{17}$$

where

$$A_0^{CCK} \equiv \phi_0 \left(\frac{M_{-1} + R_{-1} B_{-1}}{P_{-1}} \right) + \phi_0 \left[g_{-1} - \tau_{-1}^n w_{-1} n_{-1} \right] \tag{18}$$

is a time-zero constant. Defining

$$H_t^{CCK} \equiv u_{1t}c_{1t} + u_{2t}c_{2t} + u_{3t}n_t, \tag{19}$$

the PVIC can be expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t H_t^{CCK} = A_0^{CCK}. \tag{20}$$

The Ramsey problem is thus to maximize lifetime discounted utility by choosing $\{c_{1t}, c_{2t}, n_t\}$ subject to (16) and (20). The Ramsey policy that supports the optimal allocation can then be constructed using the private-sector equilibrium conditions. In particular, the Ramsey-optimal inflation process $\{\pi_t\}$ can be constructed from the sequence of intertemporal conditions (14) (lagged one period).

3 Svensson Timing

Now consider a slightly different timing, which is a straightforward adaptation of Svensson (1985) to a cash/credit environment (Svensson's (1985) model features only cash goods — that is, *all* goods are subject to the cash constraint). Here, the cash that households must use to purchase cash goods in period t is carried over from period $t-1$; it may not be adjusted in response to the realized period-t state of the economy. Thus, one may say that financial markets do *not* meet before goods markets, in contrast to the LS/CCK assumption.

3.1 Households

The household flow budget constraint in this environment reads

$$M_{t+1} - M_t + B_t - R_{t-1}B_{t-1} = (1-\tau_{t-1}^n)P_{t-1}w_{t-1}n_{t-1} - P_{t-1}c_{1t-1} - P_{t-1}c_{2t-1}, \tag{21}$$

and the cash-in-advance constraint is unchanged from the LS/CCK environment,

$$P_t c_{1t} \leq M_t. \tag{22}$$

Note well the difference between the pairs of constraints (21) and (22) and (3) and (4): here, it is M_{t+1} that the household chooses in period t for use in period $t+1$. Clearly, the only first-order condition that is different from the LS/CCK model is that on money holdings. Continuing to use the sequence of Lagrange multipliers ϕ_t/P_{t-1} and λ_t/P_t for the sequence of flow budget constraints and cash-in-advance constraints, respectively, the household first-order condition with respect to M_{t+1} is

$$-\frac{\phi_t}{P_{t-1}} + \beta E_t\left(\frac{\lambda_{t+1}}{P_{t+1}}\right) + \beta E_t\left(\frac{\phi_{t+1}}{P_t}\right) = 0. \tag{23}$$

Expression (12) still holds here, but the form useful for determining Ramsey-optimal inflation is at least superficially different. Combining (6) and (23), we have, after a few manipulations,

$$\phi_t = \frac{u_{2t}}{\pi_t} + \frac{1}{\pi_t} E_t \left[\frac{\beta(u_{1t+1} - u_{2t+1})}{\pi_{t+1}} \right]. \tag{24}$$

Using this in (12) gives us the pricing formula for a one-period risk-free nominal bond,

$$1 = R_t E_t \left[\frac{\beta \left(u_{2t+1} + R_{t+1} - 1 \right)}{u_{2t} + R_t - 1} \frac{1}{\pi_{t+1}} \right]. \tag{25}$$

This condition (lagged one period) will be the condition that pins down the Ramsey-optimal inflation rate in the Svensson (1985) timing.

Using (25), the household first-order conditions (10), (6), and (7) also imply

$$\beta E_t \left[\frac{u_{1t+1} - u_{2t+1}}{u_{2t}} \frac{1}{\pi_{t+1}} \right] = R_t - 1, \tag{26}$$

which replaces the condition $u_{1t}/u_{2t} = R_t$ from the CCK model. This condition describes the cash/credit margin (equivalently, the no-arbitrage condition between nominal bonds and money holdings), which, under the Svensson timing, is effectively an intertemporal margin: the prevailing nominal interest rate in period t affects the cash good/credit good tradeoff in period $t+1$. As in the LS/CCK environment, in a monetary equilibrium $R_t \geq 1$ must hold.

Finally, the consumption-leisure optimality condition of course continues to be given by (11).

3.2 Ramsey Allocations

The resource constraint is still (16). Again, in principle, the Ramsey planner faces $R_t \geq 1$. However, it can be shown, by virtually the same arguments as in CCK, that the Friedman Rule $R_t = 1 \; \forall t$ holds in the Svensson environment as well (provided we maintain the weak separability assumption about $u(.)$.)[2]

The Ramsey problem takes the same form as above: the Ramsey planner chooses processes $\{c_{1t}, c_{2t}, n_t\}$ in order to maximize lifetime discounted utility subject to (16) and the PVIC (the derivation of which we again suppress because it is a trivial modification of the CCK derivations), which here takes the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\beta u_{1t+1} c_{1t+1} + u_{2t} c_{2t} + u_{3t} n_t \right] = A_0^{SVEN}, \tag{27}$$

where

$$A_0^{SVEN} \equiv \phi_0 \left(\frac{M_0 + R_{-1} B_{-1}}{P_{-1}} \right) + \phi_0 \left[g_{-1} - \tau_{-1}^n w_{-1} n_{-1} \right] - \beta \phi_1 c_{1,0} \tag{28}$$

[2] Virtually the same CCK proof goes through because in the Ramsey first-order condition on c_{1t}, we pick up the term $\partial H_{t-1}^{SVEN}/\partial c_{1t}$ (define below), which is identical to $\partial H_t^{CCK}/\partial c_{1t}$. Because all of these terms are in the same, single, PVIC, except for the definitional timing, functional expressions used in proving the optimality of the Friedman Rule are the same.

Using the binding CIA constraint and the definition of inflation,

$$A_0^{SVEN} = \phi_0 \left(c_{1,0}\pi_0 + \frac{R_{-1}B_{-1}}{P_{-1}} \right) + \phi_0 \left[g_{-1} - \tau_{-1}^n w_{-1} n_{-1} \right] - \beta\phi_1 c_{1,0}. \tag{29}$$

Defining

$$H_t^{SVEN} \equiv \beta u_{1t+1} c_{1t+1} + u_{2t} c_{2t} + u_{3t} n_t, \tag{30}$$

the PVIC can be expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t H_t^{SVEN} = A_0^{SVEN}. \tag{31}$$

Note that $H_t^{SVEN} \neq H_t^{CCK}$ and $A_0^{SVEN} \neq A_0^{CCK}$. In particular, H_t^{SVEN} contains a forward-looking term about cash-good consumption, arising from the fact that money accumulated in one period is not used for cash-good consumption until the subsequent period. The Ramsey policy that supports the optimal allocation can then be constructed using the private-sector equilibrium conditions. In this environment, the Ramsey-optimal inflation process $\{\pi_t\}$ can be constructed from the sequence of intertemporal conditions (25) (lagged one period).

4 Quantitative Results

Our focus is on the dynamics of optimal policy, in particular on the dynamics of Ramsey inflation. Even more precisely, what we are interested in, as has been the entire universe of Ramsey studies to which we have referred, is the asymptotic Ramsey dynamics, meaning arbitrary initial conditions are unimportant. To make this idea as clear as possible, we limit attention to the time $t > 0$ first-order conditions of the Ramsey problem, solve numerically for the non-stochastic steady state implied by those conditions, and then assume the economy begins in that "timeless" steady state when we subject our model to business-cycle magnitude shocks to TFP and government spending.

We thus approximate our model by linearizing in levels the Ramsey first-order conditions for time $t > 0$ around the non-stochastic steady state of these conditions. Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004c), which has proven successful in a wide variety of Ramsey models (see Schmitt-Grohe and Uribe (2004a, 2004b, 2005), Chugh (2006a, 2006b), and Aruoba and Chugh (2006)). The parameter values we use are quite standard in this class of models, and we refer the reader to Appendix A for our parameter settings; because our focus is on comparing models, rather than matching models to data, the precise calibration is not critical. We of course keep all parameter values fixed across the two models to make comparisons meaningful.

We conduct 1000 simulations, each 500 periods long. To make comparisons across the two models as meaningful as possible, we use the same set of random disturbances to TFP and government spending in both models. For each simulation, we then compute first and second moments and report the medians of these moments across the 1000 simulations.

4.1 Both TFP and Government Spending Shocks

Since CCK, the precedent in much of the Ramsey literature has been to study asymptotic dynamics in the face of both supply shocks and demand shocks. Table 1 presents simulation-based first and second-moments for key Ramsey policy and allocation variables of interest under the two different timings of markets when our model economies are driven by shocks to both TFP and government spending. The top row of each panel shows our first main result: *overall inflation variability is very high under both LS/CCK timing and Svensson (1985) timing.* The mean around which inflation fluctuates in both cases is the Friedman deflation. While inflation variability is a bit lower in the Svensson environment than in the LS/CCK environment — a standard deviation of 4.58 percent (annualized) compared to 5.02 percent — we think any central banker would still consider this extreme inflation volatility. Thus, the conjecture that the Svensson (1985) timing delivers lower inflation volatility is true, but it seems quantitatively unimportant in the face of a full battery of shocks.

Regarding the other policy instruments, the labor tax rate is nearly 50 percent more variable in the Svensson environment, reflecting the somewhat smaller windfall revenues through inflation the Ramsey government is able to engineer. The Friedman rule of a zero net nominal interest rate obtains under both regimes (in these simulations, we did not impose the analytical result that a zero nominal interest rate is always optimal that obtains in both models — we simply let the approximations work, and they agree with the analytical result).

The most important difference across the two models is in the volatility of equilibrium labor — it is 57 percent more volatile in the Svensson environment than in the LS/CCK environment. The reason labor is more volatile in the Svensson environment is related to the result that the volatility of consumption is virtually no different in the two models. A given amount of price level fluctuation implies, *ceteris paribus*, higher variability of cash-good consumption if nominal money holdings are pre-determined than if they are not pre-determined. Indeed, such adverse consequences for consumption-smoothing are the basis for the conjecture that price-level variability would be undesirable in the Svensson environment. Some additional fluctuation in labor supply, which supports output, apparently can make up for this, and quantitatively this effect is sufficiently strong. We disentangle the mechanism underlying this result in the next two subsections.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x,Y)	Corr(x,Z)	Corr(x,G)
			LS/CCK timing			
$\pi-1$	-3.7675	5.0224	0.7385	-0.7309	-0.9143	0.2331
τ^n	0.1845	0.0095	0.9348	0.7803	0.9675	-0.2285
$R-1$	0	0	—	—	—	—
gdp	0.3290	0.0067	0.9292	1.0000	0.9123	0.4190
c_1	0.0238	0.0003	0.9348	0.7803	0.9675	-0.2285
c_2	0.2451	0.0029	0.9348	0.7803	0.9675	-0.2285
c_1+c_2	0.2690	0.0031	0.9348	0.7803	0.9675	-0.2285
n	0.3290	0.0028	0.8732	0.1490	-0.2601	0.9586
			Svensson timing			
$\pi-1$	-3.7985	4.5845	0.7556	-0.7585	-0.9523	—
τ^n	0.1844	0.0129	0.9148	0.3047	0.8135	-1
$R-1$	0	0	—	—	—	—
gdp	0.3290	0.0072	0.9154	1.0000	0.7970	—
c_1	0.0238	0.0003	0.9401	0.7970	1.0000	—
c_2	0.2452	0.0026	0.9401	0.7970	1.0000	—
c_1+c_2	0.2690	0.0028	0.9401	0.7970	1.0000	—
n	0.3291	0.0044	0.8732	0.3641	-0.2601	1

Table 1: Simulation-based moments, using first-order approximation, with both TFP and government spending shocks as driving force.

4.2 Government Spending Shocks

To those familiar with the basic results in the Ramsey literature, what ought to be especially striking in Table 1 is the fact that, with the exception of τ^n and n, none of the Ramsey allocation or policy variables co-vary with government spending under the Svensson (1985) timing, as the last column of the lower panel of Table 1 shows, whereas the covariations with government spending are non-zero under LS/CCK timing. The co-variances with g emerging from our simulations were different from zero in nothing greater than the eighth decimal places (which we can also observe from our computed linear decision rules), so we report these as no co-variations at all. To better understand this feature of the Ramsey equilibrium, we report in Table 2 results from simulations with the level of TFP always fixed at its mean level, so that it is only variations in government spending to which the Ramsey equilibrium is responding.

The results in Table 2 reveal our second main result: *under the Svensson (1985) timing, inflation is not used to finance pure government absorption shocks at all.* This result stands in sharp contrast to the benchmark CCK result that inflation *is* state-contingent with respect to pure government absorption shocks. As already mentioned, the intuition behind the CCK result is that (large) state-contingent movements in the price level, by varying the real returns on the government's outstanding nominal debt stock, are the least distortionary way of financing a budget shock. This part of the Ramsey mechanism in a typical cash/credit model, then, indeed does depend critically on the assumed timing of the environment. By not varying inflation following a government spending shock, the Ramsey policy calls for a more variable labor tax rate in the Svensson environment than in the LS/CCK environment, as comparison of the second rows in each panel of Table 2 shows. Note, however, that even in the basic LS/CCK framework, the inflation variability induced by variations in government spending is only about one quarter the total inflation variability induced by variations in both government spending and TFP (compare the standard deviation of inflation of 1.3721 percent in Table 2 with the standard deviation of 5.0224 percent in Table 1).

The larger fluctuations in the labor tax rate in the Svensson environment in turn lead to larger fluctuations in labor itself. We mentioned above that variations in labor are used by the Ramsey planner to support consumption-smoothing in the Svensson environment. The results here bear out this conjecture — Ramsey-optimal consumption does not vary at all. Clearly, the only way for consumption to remain unchanged in the face of pure government absorption shocks is if labor moves in a perfectly offsetting way, and the Ramsey planner uses the labor tax rate as the instrument to achieve this goal.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
			LS/CCK timing			
$\pi - 1$	-3.9276	1.3721	0.5507	0.9168	—	0.9168
τ^n	0.1847	0.0024	0.8688	-0.9999	—	-0.9999
$R - 1$	0	0	—	—	—	—
gdp	0.3290	0.0027	0.8688	1.0000	—	1.0000
c_1	0.0238	0.0001	0.8688	-1.0000	—	-1.0000
c_2	0.2452	0.0007	0.8688	-1.0000	—	-1.0000
$c_1 + c_2$	0.2690	0.0008	0.8688	-1.0000	—	-1.0000
n	0.3290	0.0027	0.8688	1.0000	—	1.0000
			Svensson timing			
$\pi - 1$	-3.9404	0	—	—	—	—
τ^n	0.1846	0.0074	0.8688	-1	—	-1
$R - 1$	0	0	—	—	—	—
gdp	0.3290	0.0043	0.8688	1	—	—-
c_1	0.0238	0	—	—	—	—
c_2	0.2452	0	—	—	—	—
$c_1 + c_2$	0.2690	0	—	—	—	
n	0.3290	0.0043	0.8688	1	—	1

Table 2: Simulation-based moments, using first-order approximation, with only government spending shocks as driving force.

4.3 TFP Shocks

It is not only pure shocks to government absorption, however, that affect the consolidated government budget constraint in equilibrium. In general, *any* shock to either demand or supply has an equilibrium impact on the budget constraint — in our primal formulation of the Ramsey problems, on the PVIC. As such, before concluding that Ramsey inflation variability in a cash/credit model depends *only* on the assumed timing of markets, we also need to know how policy responds to just TFP shocks in the two models. Table 3 presents results from these experiments. Ramsey inflation dynamics are clearly very similar across the two environments in the face of only TFP shocks, as comparison of the top row of each panel shows. Indeed, the dynamics of all Ramsey allocation and policy variables are quite similar across the two environments, the main difference being in, as already discussed above, the volatility of labor. Comparing Table 3 with Table 1, the majority of inflation volatility in the LS/CCK environment is due to TFP shocks, while in the Svensson environment, all of the volatility of inflation is due to TFP shocks.

5 Discussion

5.1 Comparing the Two Environments

When the two models are subjected to the full battery of TFP and government purchase shocks, the optimality of the Friedman Rule in both environments is helpful in understanding the result that inflation variability is quite similar in the two environments. With the Friedman Rule ($R_t = 1$) in place in the Svensson environment, the cash/credit optimality condition (26) reduces to

$$\beta E_t \left[\frac{u_{1t+1} - u_{2t+1}}{u_{2t}} \frac{1}{\pi_{t+1}} \right] = 0. \tag{32}$$

In effect, then, the Ramsey allocation features $u_{1t} = u_{2t}$ in the Svensson environment.[3] Using $R_t = 1$ and this "approximate result" $u_{1t} = u_{2t}$ in the condition that determines inflation, condition (25), we have

$$1 = R_t E_t \left[\frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right]. \tag{33}$$

But this condition is identical to condition (14), which determines inflation in the LS/CCK environment. With utility weakly separable in (c_1, c_2) and n, and *given* that the consumption allocations are quite similar in the two models, it should be expected that Ramsey inflation would be quite similar. As the results in Table 1 show, ensuring that consumption allocations in the Svensson environment are close to those in the LS/CCK environment requires households to vary their work

[3]This is of course not literally correct, because the cash/credit optimality condition in the Svensson environment is an expectational condition — but we think this is nonetheless a useful way of understanding the results.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x,Y)	Corr(x,Z)	Corr(x,G)
			LS/CCK timing			
$\pi-1$	-3.7796	4.8495	0.7556	-0.9519	-0.9523	—
τ^n	0.1845	0.0093	0.9401	0.9999	1.0000	—
$R-1$	0	0	—	—	—	—
gdp	0.3290	0.0061	0.9400	1.0000	0.9999	—
c_1	0.0238	0.0003	0.9401	0.9999	1.0000	—
c_2	0.2451	0.0028	0.9401	0.9999	1.0000	—
c_1+c_2	0.2690	0.0031	0.9401	0.9999	1.0000	—
n	0.3290	0.0008	0.9401	-0.9999	-1.0000	—
			Svensson timing			
$\pi-1$	-3.7985	4.5845	0.7556	-0.7585	-0.9523	—
τ^n	0.1844	0.0107	0.9401	0.9999	1	—
$R-1$	0	0	—	—	—	—
gdp	0.3290	0.0056	0.9400	1.0000	0.9999	—
c_1	0.0238	0.0003	0.9401	0.9999	1.0000	—
c_2	0.2452	0.0026	0.9401	0.9999	1.0000	—
c_1+c_2	0.2690	0.0028	0.9401	0.9999	1.0000	—
n	0.3290	0.0013	0.9401	-0.9999	-1.0000	—

Table 3: Simulation-based moments, using first-order approximation, with only TFP shocks as driving force.

effort more. Given the typical (large) macro-style calibration for the elasticity of labor supply that we use, such time-variation in labor is tolerable to the representative household.

5.2 Larger Implications for the Ramsey Literature

We view our results as demonstrating that the bias in favor of inflation variability in a baseline cash/credit Ramsey monetary model does not stem from the assumed LS/CCK timing. Granted, the inflation response *to a pure government absorption shock* is quite different in the two environments, but the response to TFP shocks is quite similar. Because both TFP and government purchase disturbances — as well as a host of many other shocks that can be classified as either "demand" or "supply" shocks — are surely important in reality, the Ramsey framework offers a unified way to think about what types of shocks and, more generally, what types of frictions in the environment might or might not make inflation stability important as a policy goal.

The logic behind this assertion is the following. The basic way of understanding inflation variability in any Ramsey-based model is that it is used (benevolently) to affect consumers' portfolios. More specifically, its use makes bond Euler equations *not* hold with equality ex-post. Of course, Euler equations in general do not hold with equality after the realization of shocks — but the magnitude of these bond Euler equation "errors" are very large in the basic Ramsey environment. Under Svensson timing, hitting the Euler equation in this way following pure government absorption shocks would have the consequence that consumption would be highly variable, simply because consumers cannot adjust their nominal money holdings in the same state-contingent manner that the Ramsey planner can adjust the price level. The benevolent Ramsey government thus avoids such variability.

However, most conventional thinking about why inflation variability may be undesirable in practice regards its effects on within-period, static, relative prices, not its ex-post effects on bond Euler equations. Indeed, a basic lesson that New Keynesian sticky-price models have taught the profession is that when static relative prices are distorted by variations in inflation, the welfare consequences can be large. Thus, another way we find illuminating to understand the results we obtain is that no matter what the timing of markets, there is no within-period relative price distortion caused by state-contingent inflation. This point is made cleanly by the result that optimal inflation variability is high in the face of TFP shocks no matter the assumed timing of markets. That the Ramsey planner does not stabilize inflation following TFP shocks no matter the timing of markets is evidence that timing *per se* does not create undesirable static relative price distortions due to inflation.

The natural candidate relative price to be distorted by state-contingent variations in actual inflation is that between cash and credit goods. However, up to a given a nominal interest rate, the static relative price between cash goods and credit goods is identically one because of their unit marginal rate of transformation and the assumption that they are both obtained by consumers in Walrasian fashion.[4] Clearly, then, the timing of markets has nothing to do with whether or how inflation (or anything else, for that matter) alters this relative price. It is this primitive feature of the cash/credit model — that it features a unit marginal rate of transformation and hence an efficient marginal rate of substitution of unity — that we think makes the Ramsey model a powerful diagnostic tool for uncovering features and frictions in the economy that may actually be important for stabilizing inflation. If one introduces to the basic Ramsey environment friction X and the optimal policy features inflation stability, one has cleanly isolated friction X as one that makes inflation stability an important goal to pursue. One can read the recent outburst of Ramsey studies of optimal fiscal and monetary policy — both those that feature a cash/credit structure at their core as well as those that do not — as very much in this spirit. Our results show that timing assumptions do not blur the main messages that emerge from such studies.

6 Conclusion

We investigated the quantitative importance, in a cash/credit model, of the precise temporal ordering of markets within a period — whether financial markets meet before or after goods markets — for the basic Ramsey prescription of inflation volatility and found that it matters for how the government ought to respond to pure government absorption shocks but not for how it ought to respond to technology shocks. Our focus on the volatility (or potential lack thereof) in a baseline Ramsey model is driven by the growing number of studies that use the Ramsey framework of jointly-optimal fiscal and monetary policy as a laboratory for testing what frictions in the economy may be important for making inflation stability an important goal of policy. Those that are skeptical about the recommendation of using inflation policy to offset pure fiscal shocks should be comforted by the finding that this result is not robust across all cash/credit timing conventions. At the same time, our finding that the optimal policy response to a key shock — the TFP shock — is not sensitive to this particular timing assumption inspires some confidence regarding the interpretation of results obtained in richer Ramsey models that are being built around a cash/credit core.

[4] That is, with the exception of the cash-in-advance constraint, the fundamental environments in which cash goods and credit goods are transacted are identical. One could say that the literature on the "search microfoundations" of money (For some of the latest developments, see Lagos and Wright (2005)), takes issue with this latter assumption.

A Parameterization

The time unit in our model is one quarter, so we set the subjective time discount factor to $\beta = 0.99$. We use a period utility function

$$u(c_t, n_t) = \frac{(c_t(1-n_t)^\zeta)^{1-\sigma} - 1}{1-\sigma}, \tag{34}$$

with the consumption index a CES aggregate of cash goods and credit goods,

$$c_t = \Big((1-\gamma)c_{1t}^\phi + \gamma c_{2t}^\phi\Big)^{1/\phi}. \tag{35}$$

We set $\sigma = 1$ so that preferences are separable in leisure and consumption. In the consumption aggregator, we use $\phi = 0.79$ and $\gamma = 0.62$, as estimated by Siu (2004). The parameter ζ is calibrated so that in the model with perfect competition in both product and labor markets, consumers spend a fraction $n = 0.30$ of their time working in the deterministic steady-state of the Ramsey allocation. The value that we need turns out to be $\zeta = 2.27$, and we hold this value constant as we move to the imperfectly competitive environments with various combinations of sticky prices and sticky wages.

The exogenous productivity and government spending shocks follow AR(1) processes in logs,

$$\ln(z_t) = \rho_z \ln(z_{t-1}) + \epsilon_t^z, \tag{36}$$

$$\ln(g_t) = (1-\rho_g)\ln(\bar{g}) + \rho_g \ln(g_{t-1}) + \epsilon_t^g, \tag{37}$$

where $\bar{g}$ denotes the steady-state level of government spending, which we calibrate in the model with perfect competition in both product and labor markets to constitute 20 percent of steady-state output in the Ramsey allocation. The resulting value is $\bar{g} = 0.06$, which we hold constant as we vary the degrees of imperfect competition and nominal rigidities in goods and labor markets. The innovations ϵ_t^z and ϵ_t^g are distributed $N(0, \sigma_{\epsilon^z}^2)$ and $N(0, \sigma_{\epsilon^g}^2)$, respectively, and are independent of each other. We choose parameters $\rho_z = 0.95$, $\rho_g = 0.97$, $\sigma_{\epsilon^z} = 0.007$, and $\sigma_{\epsilon^g} = 0.03$ in keeping with the RBC literature and Chari, Christiano, and Kehoe (1991). Also regarding policy, we assume that the steady-state government debt-to-GDP ratio (at an annual frequency) is 0.5, in line with evidence for the U.S. economy and with the calibrations of Schmitt-Grohe and Uribe (2004b) and Siu (2004).

References

Arseneau, David M. and Sanjay K. Chugh. 2007. "Optimal Fiscal and Monetary Policy with Costly Wage Bargaining." International Finance Discussion Paper no. 893 , Board of Governors of the Federal Reserve System.

Aruoba, S. Boragan and Sanjay K. Chugh. 2006. "Optimal Fiscal and Monetary Policy When Money is Essential." International Finance Discussion Paper no. 880 , Board of Governors of the Federal Reserve System.

Chari, V.V., Lawrence Christiano, and Patrick Kehoe. 1991. "Optimal Fiscal and Monetary Policy: Some Recent Results." *Journal of Money, Credit, and Banking,* Vol. 23, pp. 519-539.

Chugh, Sanjay K. 2006a. "Optimal Fiscal and Monetary Policy with Sticky Wages and Sticky Prices." *Review of Economic Dynamics,* Vol. 9, pp. 683-714.

Chugh, Sanjay K. 2006b. "Optimal Inflation Persistence: Ramsey Taxation with Capital and Habits." *Journal of Monetary Economics.* Forthcoming.

Lagos, Ricardo and Randall Wright. 2005. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy,* Vol. 113, pp. 463-484.

Lucas, Robert E. Jr. and Nancy L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics,* Vol. 12, pp. 55-93.

Schmitt-Grohe, Stephanie and Martin Uribe. 2004a. "Optimal Fiscal and Monetary Policy Under Imperfect Competition." *Journal of Macroeconomics,* Vol. 26, pp. 183-209.

Schmitt-Grohe, Stephanie and Martin Uribe. 2004b. "Optimal Fiscal and Monetary Policy Under Sticky Prices." *Journal of Economic Theory,* Vol. 114, pp. 198-230.

Schmitt-Grohe, Stephanie and Martin Uribe. 2004c. "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function." *Journal of Economic Dynamics and Control,* Vol. 28, pp. 755-775.

Schmitt-Grohe, Stephanie and Martin Uribe. 2005. "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model." NBER Macroeconomics Annual 2005.

Siu, Henry E. 2004. "Optimal Fiscal and Monetary Policy with Sticky Prices." *Journal of Monetary Economics,* Vol. 51, pp. 576-607.